1.1 Rational Expressions

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	1. Algebra & Functions
Topic	1.1 Rational Expressions
Difficulty	Medium

Time allowed: **60**

Score: **/53**

Percentage: **/100**

Question 1

Simplify

(i) $\frac{x}{2x}$

(ii) $\frac{x+1}{x(x+1)}$

(iii) $\frac{6x+12}{x^2+2x}$

[4 marks]

Question 2

(a) Simplify fully $\frac{2x^2+6x}{x^3+3x^2}$

[3 marks]

Question 2

(b) Simplify fully $\frac{x+4}{x^3} \times \frac{x^2+2x}{x+4}$

[3 marks]

Question 2

(c) Simplify fully $\frac{x^2+4x}{3x+6} \div \frac{2x+8}{x+2}$

[3 marks]

Question 3

The function $\mathrm{f}(x)$ is given by

$$\mathrm{f}(x) = 3x^3 - 5x^2 - 4x + 4$$

(a) Show that $\mathrm{f}\left(\frac{2}{3}\right) = 0$.

[2 marks]

Question 3

(b) Hence write down a factor of $\mathrm{f}(x)$.

[1 mark]

Question 3

(c) Fully factorise $\mathrm{f}(x)$.

[3 marks]

Question 3

(d) Write down the solutions to the equation $\mathrm{f}(x) = 0$.

[2 marks]

Question 4

(a) Show that $(2x - 3)$ is a factor of $2x^3 - 13x^2 + 23x - 12$.

[2 marks]

Question 4

(b) Fully factorise $2x^3 - 13x^2 + 23x - 12$.

[2 marks]

Question 4

(c) Find all the real solutions to $2x^3 - 13x^2 + 23x - 12 = 0$.

[2 marks]

Question 5

Given that $(2x - 1)$ is a factor of $2x^3 + x^2 - 25x + a$ find the value of a.

[2 marks]

Question 6

(a) Work out $(x^3 + 3x^2 - 2x + 4) \div (x + 1)$.

[3 marks]

Question 6

(b) Work out $\dfrac{2x^3 - 4x + 3}{x - 2}$

[3 marks]

Question 7

Given $(x^2 + 8x - 4) \div (x - 3) = x + 11 + \dfrac{29}{x - 3}$

(a) (i) Write down the divisor.
(ii) Write down the quotient.
(iii) Write down the remainder.

[3 marks]

Question 7

(b) (i) Write down the degree of $x^2 + 8x - 4$.

(ii) Write down the degree of $x - 3$.

(iii) Explain why you would expect the quotient to be of degree 1 in this case.

[3 marks]

Question 8

One of the three algebraic fractions below is improper ('top-heavy').

$$\frac{x+2}{x^2+2} \qquad \frac{x}{x+2} \qquad \frac{1}{x+2}$$

Identify which fraction is improper and write it in the form $A + \frac{B}{x+2}$, where A and B are integers to be found.

[3 marks]

Question 9

(a) Simplify fully $\dfrac{x^3 - 2x^2 - 8x}{x - 4}$

[2 marks]

Question 9

(b) Hence solve the equation $\dfrac{x^3 - 2x^2 - 8x}{x - 4} = x^2 + 10x + 16.$

[3 marks]

Question 10

It is given that

$$\frac{f(x)}{x+2} = 3x + 4 - \frac{2}{x+2}$$

Find $f(x)$.

[2 marks]

Question 11

The result of dividing $x^2 + ax - 5$ by $(x+1)$ is $x + 3 + \frac{d}{x+1}$.

Find the values of a and d.

[2 marks]

1.2 Functions

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	1. Algebra & Functions
Topic	1.2 Functions
Difficulty	Medium

Time allowed: **60**

Score: **/53**

Percentage: **/100**

Question 1

State whether the following mappings are one-to-one, many-to-one, one-to-many or many-to-many.

(i) $\mathrm{f}: x \mapsto x^2$
(ii) $\mathrm{f}: x \mapsto 3x + 1$
(iii) $\mathrm{f}: x \mapsto (x + 1)^3$
(iv) $\mathrm{f}: x \mapsto \pm\sqrt{x}$

[4 marks]

Question 2

The function $\mathrm{f}(x)$ is defined as

$$\mathrm{f}(x) = x^2 + 2x - 3 \qquad x \in \mathbb{R}$$

(a) Sketch the graph of $y = \mathrm{f}(x)$, giving the coordinates of any points where the graph intercepts the coordinate axes and the coordinates of the turning point.

[3 marks]

Question 2

(b) Write down the range of $\mathrm{f}(x)$.

[1 mark]

Question 3

The function $\mathrm{f}(x)$ is defined as

$$\mathrm{f}(x) = x^2 - 4 \qquad x \geq 0$$

(a) Work out the range of $\mathrm{f}(x)$.

[2 marks]

Question 3

(b) If the domain of $\mathrm{f}(x)$ is changed to $x \leq 0$, what is the range of $\mathrm{f}(x)$?

[1 mark]

Question 4

The functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are defined as follows

$$\mathrm{f}(x) = x^2 \qquad x \in \mathbb{R}$$
$$\mathrm{g}(x) = 4x - 3 \qquad x \in \mathbb{R}$$

(a) Write down the range of $\mathrm{f}(x)$.

[1 mark]

Question 4

(b) Find (i) $\mathrm{fg}(x)$
(ii) $\mathrm{gf}(x)$

[4 marks]

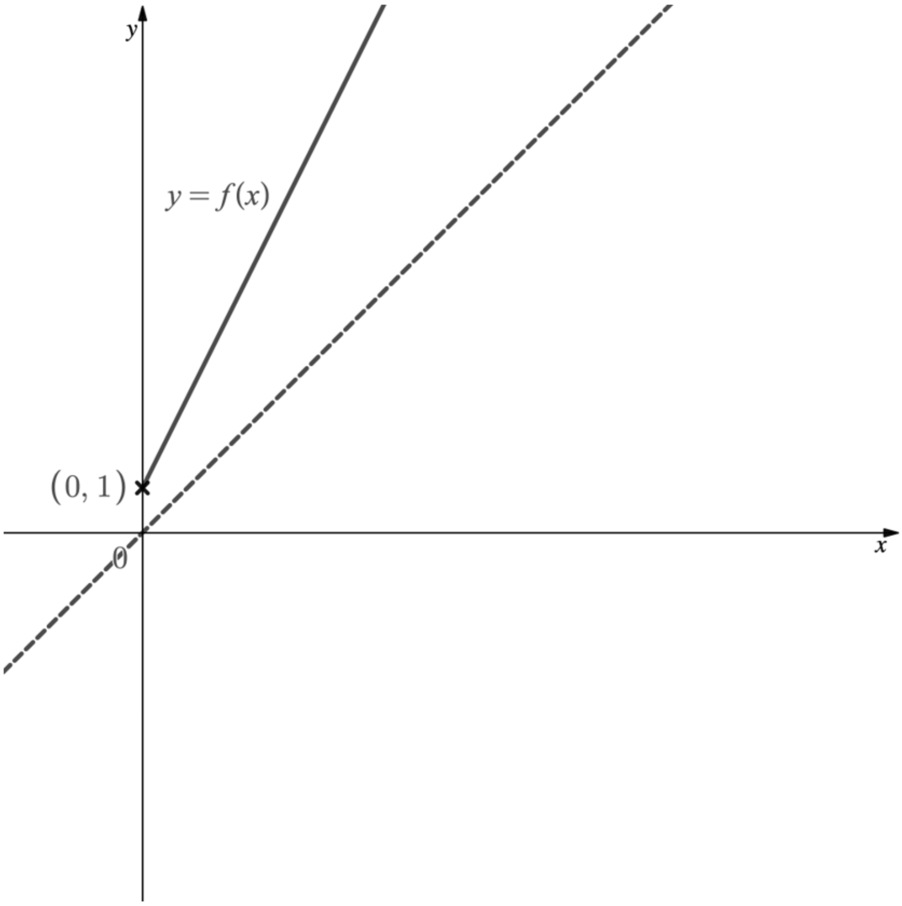
Question 4

(c) Solve the equation $\mathrm{f}(x) = \mathrm{g}(x)$.

[2 marks]

Question 5

The graph of $y = \mathrm{f}(x)$ is shown below.



(a) (i) Use the graph to write down the domain and range of $\mathrm{f}(x)$.

(ii) Given that the point $(1, 1)$ lies on the dotted line, write down the equation of the line.

[3 marks]

Question 5

(b) On the diagram above sketch the graph of $y = \mathrm{f}^{-1}(x)$.

[2 marks]

Question 6

(a) On the same axes, sketch the graphs of $y = \mathrm{f}(x)$ and $y = |\mathrm{g}(x)|$ where

$$\mathrm{f}(x) = (x+2)^2 \qquad x \in \mathbb{R}$$
$$\mathrm{g}(x) = 2x + 4 \qquad x \in \mathbb{R}$$

Label the points at which the graphs intersect the coordinate axes.

[3 marks]

Question 6

(b) Solve the equation $\mathrm{f}(x) = |\mathrm{g}(x)|$.

[2 marks]

Question 7

The function $f(x)$ is defined as

$$f: x \mapsto \frac{x^2+1}{x^2} \qquad x \in \mathbb{R},\ x \neq 0$$

(a) Show that $f(x)$ can be written in the form

$$f: x \mapsto 1 + \frac{1}{x^2}$$

[2 marks]

Question 7

(b) Explain why the inverse of $f(x)$ does not exist and suggest an adaption to its domain so the inverse does exist.

[2 marks]

Question 7

(c) The domain of $f(x)$ is changed to $x > 0$.
Find an expression for $f^{-1}(x)$ and state its domain and range.

[4 marks]

Question 8

Solve the equation $|4x + 2| = 5$.

[3 marks]

Question 9

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = \frac{1}{2}(4x - 3) \qquad x \in \mathbb{R}$$
$$g(x) = 0.5x + 0.75 \qquad x \in \mathbb{R}$$

(a) Find (i) $fg(x)$
(ii) $gf(x)$

[3 marks]

Question 9

(b) Write down $\mathrm{f}^{-1}(x)$ and state its domain and range.

[3 marks]

Question 10

The functions $\mathrm{f}(x)$, $\mathrm{g}(x)$ are defined as follows

$$\mathrm{f}(x) = |x - 9| \qquad x \in \mathbb{R}$$
$$\mathrm{g}(x) = x^2 \qquad x \in \mathbb{R}$$

(a) Sketch the graph of $y = \mathrm{fg}(x)$, stating the coordinates of all points where the graph intercepts the coordinate axes.

[4 marks]

Question 10

(b) (i) How many solutions are there to the equation $\mathrm{fg}(x) = 5$?
(ii) How many solutions are there to the equation $\mathrm{fg}(x) = 9$?

[2 marks]

Question 10

(c) Write down the solutions to the equation $\mathrm{fg}(x) = 0$.

[2 marks]

1.3 Combinations of Transformations

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	1. Algebra & Functions
Topic	1.3 Combinations of Transformations
Difficulty	Medium

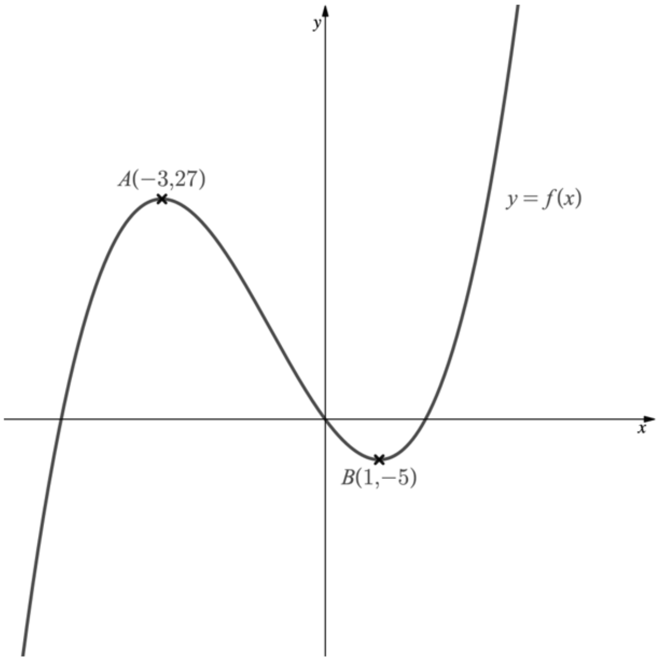
Time allowed: **60**

Score: **/47**

Percentage: **/100**

Question 1

The diagram below shows the graph of $y = \mathrm{f}(x)$.
The stationary points are marked on the diagram.



On separate diagrams, sketch the graphs with equation

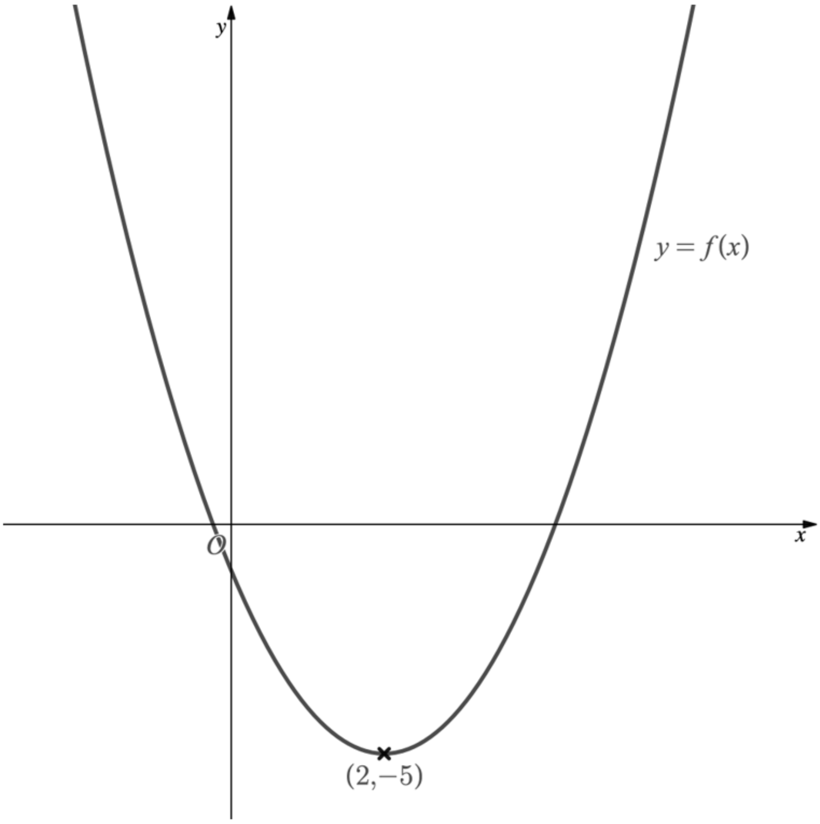
(i) $y = 2\mathrm{f}(x) - 4$
(ii) $y = \mathrm{f}(x+1) + 3$

On each diagram, state the coordinates of the images of the points A and B under the given transformation.

[4 marks]

Question 2

The turning point on the graph of $y = \mathrm{f}(x)$ has coordinates $(2, -5)$ as shown on the diagram below.



(i) On the diagram above sketch the graph of $y = |\mathrm{f}(x)| + 1$ and state the coordinates of the turning point.

(ii) State the distance between the turning points on the graphs of $y = \mathrm{f}(x)$ and $y = |\mathrm{f}(x)| + 1$.

[4 marks]

Question 3

Describe, in order, a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ onto the following graphs:

(i) $y = 3\mathrm{f}(x+2)$,
(ii) $y = \mathrm{f}(-x) - 1$.

[3 marks]

Question 4

Given that $\mathrm{f}(x) = 3x^2 - 2x$ find an expression for $\mathrm{g}(x)$, where $\mathrm{g}(x)$ is obtained by applying the following sequence of transformations to $\mathrm{f}(x)$.

1. Translation by $\binom{2}{0}$
2. Vertical stretch of scale factor 4
3. Translation by $\binom{0}{-3}$

[4 marks]

Question 5

(a) (i) Sketch the graph of $y = \mathrm{p}(x)$, where $\mathrm{p}(x) = 3x - 4$.

(ii) On the same set of axes, sketch the graph of $y = p^{-1}(x)$.

Label the coordinates of the points where each graph crosses the coordinate axes.

[4 marks]

Question 5

(b) (i) Find an expression for $\mathrm{p}^{-1}(x)$.

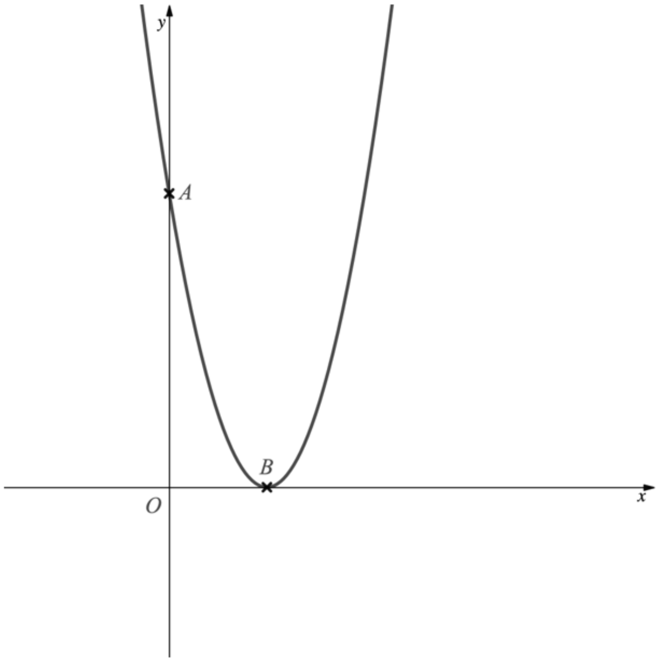
(ii) Find an expression for $\frac{1}{9}[\mathrm{p}(x) + 16]$.

(iii) What can you deduce about the sequence of transformations given by $\frac{1}{9}[\mathrm{p}(x) + 16]$?

[4 marks]

Question 6

The equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (x - a)^2$, with $a > 1$, is shown below.



The points A and B are the points where the graph intercepts the coordinate axes.

(a) Write down, in terms of a, the coordinates of A and B.

[2 marks]

Question 6

(b) Sketch the graph of $y = -\mathrm{f}(-x)$, labelling the images of the points A and B and stating their coordinates in terms of a.

[3 marks]

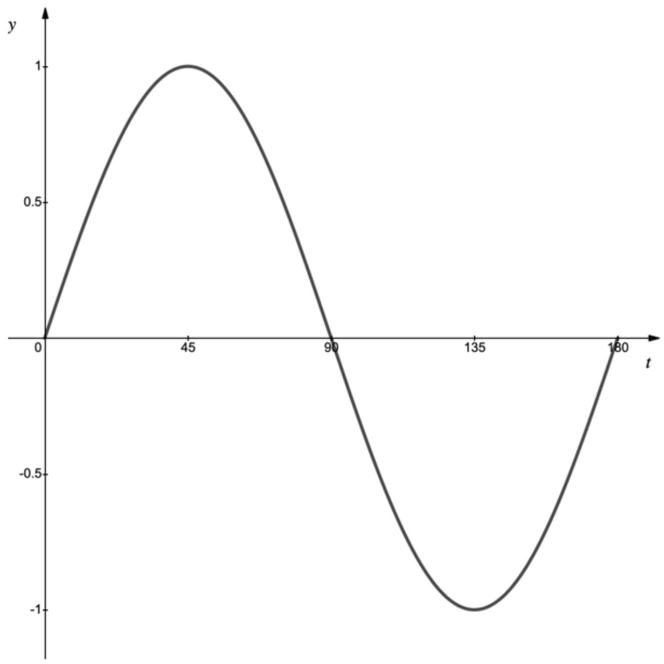
Question 6

(c) Write down the value of a such that the point A is three times as far from the origin as the point B.

[1 mark]

Question 7

The diagram shows the graph of $y = \mathrm{f}(t)$, where $\mathrm{f}(t) = \sin 2t,\ 0° \leq x \leq 180°$.



(a) (i) Write down the maximum value of y when $y = 3\mathrm{f}(t)$.

(ii) Write down the first value of t for which this maximum occurs.

[2 marks]

Question 7

(b) (i) Write down the minimum value of y when $y = 5\mathrm{f}(t + 30°)$.

(ii) Write down the first value of t for which this minimum occurs.

[2 marks]

Question 7

(c) Find, in terms of $\mathrm{f}(t)$, the combination of transformations that would map the graph of $y = \mathrm{f}(t)$ onto the graph of $y = 2 + \sin t,\ 0° \le x \le 180°$.

[2 marks]

Question 8

The function $\mathrm{f}(x)$ is to be transformed by a sequence of functions, in the order detailed below:

1. A horizontal stretch by scale factor 2
2. A reflection in the x-axis
3. A translation by $\binom{0}{2}$

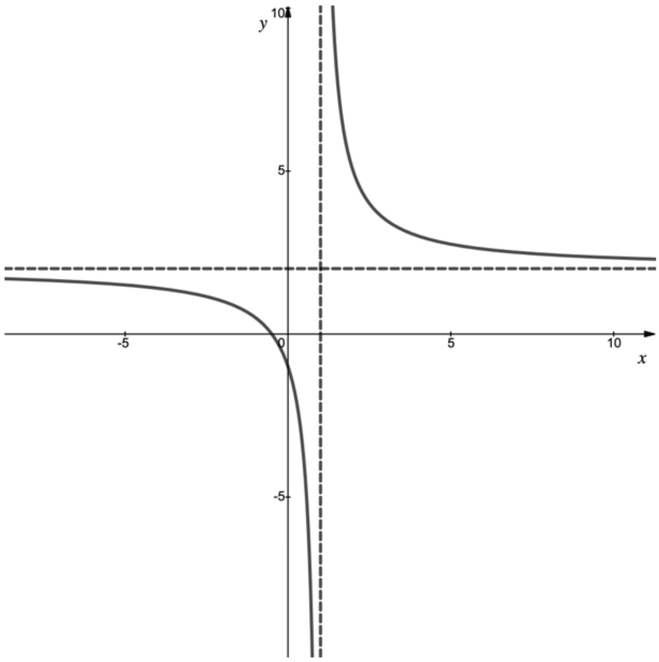
Write down an expression for the combined transformation in terms of $\mathrm{f}(x)$.

[3 marks]

Question 9

The diagram below shows the graph of $y = \mathrm{g}(x)$ where

$$\mathrm{g}(x) = \frac{2x+1}{x-1}, \qquad x \neq 1$$



(a) Write down the equations of the two asymptotes.

[2 marks]

Question 9

(b) Determine the equations of the two asymptotes on the graph of $y = \mathrm{g}(2x) - 3$.

[2 marks]

Question 9

(c) Determine the range of $|\mathrm{g}(3x) - 2|$.

[2 marks]

Question 10

The point with coordinates $(1\,, -4)$ is a stationary point on the graph with equation $y = \mathrm{h}(x)$.
Determine the coordinates of the stationary point on the graphs with the following equations:

(i) $y = 2\mathrm{h}(x-1)$,
(ii) $y = -\mathrm{h}(x+1) + 2$,
(iii) $y = |\mathrm{h}(3x) + 2|$.

[3 marks]

2.1 Reciprocal & Inverse Trigonometric Functions

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	2. Trigonometry
Topic	2.1 Reciprocal & Inverse Trigonometric Functions
Difficulty	Medium

Time allowed: **60**

Score: **/47**

Percentage: **/100**

Question 1

(a) Use the definitions of the secant, cosecant and cotangent functions to show that

$$\sec\theta \cot\theta \equiv \operatorname{cosec}\theta.$$

[2 marks]

Question 1

(b) Hence solve, in the range $0 \le \theta < 2\pi$, the equation

$$\sec\theta \cot\theta = -2.$$

[3 marks]

Question 2

(a) Show that the equation

$$3 - \sec\theta = \frac{2}{\sec\theta}$$

can be rewritten in the form

$$(\sec\theta - 2)(\sec\theta - 1) = 0.$$

[2 marks]

Question 2

(b) Hence solve, in the range $0 \leq \theta \leq 2\pi$, the equation

$$3 - \sec\theta = \frac{2}{\sec\theta}.$$

[4 marks]

Question 3

(a) Using the double angle formula $\sin 2A \equiv 2\sin A\cos A$, show that the equation

$$\sec x \operatorname{cosec} x - 5 = \operatorname{cosec} 2x$$

can be rewritten in the form

$$\operatorname{cosec} 2x = 5.$$

[3 marks]

Question 3

(b) Hence solve, in the range $0 \leq x \leq 2\pi$, the equation

$$\sec x \operatorname{cosec} x - 5 = \operatorname{cosec} 2x$$

giving your answers correct to 3 significant figures.

[3 marks]

Question 4

(a) Show that the equation

$$\tan^2 x = 6 \sec x - 10$$

can be rewritten in the form

$$(\sec x - 3)^2 = 0.$$

[3 marks]

Question 4

(b) Hence solve, in the range $0 \leq x \leq 2\pi$, the equation

$$\tan^2 x = 6 \sec x - 10$$

giving your answers correct to 3 significant figures.

[3 marks]

Question 5

Given that x satisfies the equation $\arccos x = k$, where $0 < k < \frac{\pi}{2}$,

(i) state the range of possible values of x,
(ii) express both $\sin k$ and $\tan k$ in terms of x.

[5 marks]

Question 6

(a) Prove that for $0 \leq x \leq 1$, $\arcsin x = \arccos \sqrt{1 - x^2}$.

[4 marks]

Question 6

(b) Explain why this is not true for $-1 \leq x < 0$.

[2 marks]

Question 7

(i) Sketch, in the interval $-2\pi \leq \theta \leq 2\pi$, the graph of $y = 3 + 2\,\text{cosec}\,\theta$, include asymptotes and label the coordinates of all maximum and minimum points.

(ii) Hence, deduce the number of solutions to the equation $3 + 2\,\text{cosec}\,\theta = \frac{1}{2}$ in the interval $-2\pi \leq \theta \leq 2\pi$.

[5 marks]

Question 8

The function f is defined as $\mathrm{f}(x) = \arccos x, -1 \le x \le 1$, and the function g is such that $\mathrm{g}(x) = \mathrm{f}(3x)$.

(a) Sketch the graph of $y = \mathrm{f}(x)$ and state the range of f.

[3 marks]

Question 8

(b) Sketch the graph of $y = \mathrm{g}(x)$ and state the domain of g.

[3 marks]

Question 8

(c) Find the inverse function $\mathrm{g}^{-1}(x)$ and state its domain.

[2 marks]

2.2 Compound & Double Angle Formulae

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	2. Trigonometry
Topic	2.2 Compound & Double Angle Formulae
Difficulty	Medium

Time allowed: **70**

Score: **/55**

Percentage: **/100**

Question 1

Prove by a counter-example that $\sin(A+B) = \sin A + \sin B$ is **not** true in general.

[2 marks]

Question 2

(a) Express $\tan(210°)$ in terms of $\tan(180°)$ and $\tan(30°)$.

[2 marks]

Question 2

(b) Hence show that $\tan(210°) = \dfrac{\sqrt{3}}{3}$.

[2 marks]

Question 3

(a) Starting with the identity

$$\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$$

and using the substitution $B = A$, show that $\cos 2A \equiv \cos^2 A - \sin^2 A$.

[2 marks]

Question 3

(b) Hence, or otherwise, show that $\cos 2A \equiv 1 - 2\sin^2 A$.

[2 marks]

Question 4

(a) Using an appropriate trigonometric identity, show that

$$R\sin(\theta + \alpha) \equiv R\cos\alpha\sin\theta + R\sin\alpha\cos\theta$$

where R and α are constants.

[2 marks]

Question 4

(b) Hence show that $3\sin\theta + 2\cos\theta = \sqrt{13}\sin(\theta + 0.588)$.

[3 marks]

Question 5

Use appropriate double angle formulae to solve the following equations in the given intervals.

(a) $\cos^2\theta - \sin^2\theta = \frac{1}{2}$ $\quad -\pi \leq \theta \leq \pi$

[5 marks]

Question 5

(b) $4\sin x\cos x = -\sqrt{3}$ $\quad 0 \leq x \leq \pi$

[5 marks]

Question 6

Show that

$$\frac{5\sin 2x}{\tan x} \equiv 10\cos^2 x \qquad x \neq \frac{k\pi}{2}$$

[3 marks]

Question 7

(a) (i) Show that $R\cos(x+\alpha) \equiv R\cos\alpha\cos x - R\sin\alpha\sin x$, where R and α are constants.

(ii) Use your result from part (i) to show that $\cos x - \sqrt{3}\sin x \equiv 2\cos\left(x+\frac{\pi}{3}\right)$.

[4 marks]

Question 7

(b) Hence solve the equation $\cos x - \sqrt{3}\sin x = 1$ for $0 \leq x \leq 2\pi$.

[3 marks]

Question 8

(a) Using the identities

$$\sin(A + B) \equiv \sin A \cos B + \sin B \cos A \text{ and}$$

$$\cos 2A \equiv 1 - 2\sin^2 A$$

show that $\sin 3A \equiv 3\sin A - 4\sin^3 A$

[5 marks]

Question 8

(b) Hence, or otherwise, solve the equation

$$3\sin\theta - 4\sin^3\theta = \frac{1}{2} \qquad\qquad -\pi \leq \theta \leq \pi$$

[4 marks]

Question 9

(a) Show that $5\sin\theta + 12\cos\theta$ can be written in the form $R\sin(\theta + \alpha°)$ where $R > 0$ and $0° < \alpha < 90°$

[4 marks]

Question 9

(b) Sketch the graph of $y = 5\sin x + 12\cos x$ for $0° \leq x \leq 360°$.
Label any points where the graph intercepts the coordinate axes.

[4 marks]

Question 10

Show that $2\,\text{cosec}\,2A \equiv \text{cosec}\,A\sec A$.

[3 marks]

2.3 Further Trigonometric Equations

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	2. Trigonometry
Topic	2.3 Further Trigonometric Equations
Difficulty	Medium

Time allowed: **70**

Score: **/56**

Percentage: **/100**

Question 1

Solve the equation $\operatorname{cosec} 2\theta = 2$ for $0° \leq \theta \leq 180°$.

[3 marks]

Question 2

Given that

$$\tan(3A° - 30°) = -\frac{\sqrt{3}}{3}$$

find the values of A such that $-120° \leq A° \leq 120°$.

[3 marks]

Question 3

Solve the equation

$$\frac{\sin x}{\sec x} = \frac{1}{4}, \qquad -\pi \leq x \leq \pi$$

[4 marks]

Question 4

(a) On the same axes, sketch the graphs of $y = \arcsin x$ and $y = \arccos x$, where x is measured in radians.
Label any points where the graphs intersect the coordinate axes and each other.

[4 marks]

Question 4

(b) Hence, or otherwise find the only solution to $\cos(\arcsin x) = x$.

[2 marks]

Question 5

(a) Use the identity

$$R\sin(A + B) \equiv R\cos B \sin A + R\sin B \cos A$$

to show that

$$3\sin\theta + 4\cos\theta$$

can be written as

$$5\sin(\theta + \alpha) \qquad \text{where } \alpha = \tan^{-1}\left(\frac{4}{3}\right)$$

[4 marks]

Question 5

(b) Hence, or otherwise, solve the equation $3\sin\theta + 4\cos\theta = 1$ for $0 \leq \theta \leq \pi$.
Give your answers to three significant figures.

[3 marks]

Question 5

(c) Write down the maximum value of $3\sin\theta + 4\cos\theta$ and state the first positive value of θ for which it occurs. Give your value of θ to three significant figures.

[2 marks]

Question 6

(a) Show that the equation $3\tan^2 x = 18 - 2\sec x$ can be written as

$$3\sec^2 x + 2\sec x - 21 = 0$$

[2 marks]

Question 6

(b) Hence, or otherwise, solve the equation

$$3\tan^2 x = 18 - 2\sec x, \quad -\pi \leq x \leq \pi$$

Give your answers to three significant figures.

[4 marks]

Question 7

Solve the equation

$$\cos 2\theta = \cos\theta - 1 \qquad\qquad -\pi \le \theta \le \pi$$

State your answers as multiples of π.

[5 marks]

Question 8

(a) Write down the domain and range for the function

$$\mathrm{f}(x) = \arcsin x$$

[2 marks]

Question 8

(b) Solve the equation

$$[\mathrm{f}(x)]^2 = \frac{\pi^2}{16}$$

[3 marks]

Question 9

(a) Use a small angle approximation to estimate the solution to the equation

$$\operatorname{cosec}\theta = 10$$

[2 marks]

Question 9

(b) Solve the equation $\operatorname{cosec}\theta = 10 \quad 0 < \theta < \frac{\pi}{2}$.

Give your answer to six decimal places.

[2 marks]

Question 9

(c) Find the percentage error, to two significant figures, in the approximation from part (a) compared to your answer in part (b).

[2 marks]

Question 10

(a) Sketch the graph of $y = \cot^2\theta$ for $-2\pi \le \theta \le 2\pi$.

[3 marks]

Question 10

(b) By adding three lines to your graph demonstrate how the equation

$$\cot^2\theta = k \qquad -2\pi \le \theta \le 2\pi$$

where k is a constant has either 0, 4 or 8 real solutions.

[2 marks]

Question 11

Solve the equation

$$\cot^2\theta = \sec^2\theta - 1, \qquad 0° \leq \theta \leq 360°$$

[4 marks]

2.4 Trigonometric Proof

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	2. Trigonometry
Topic	2.4 Trigonometric Proof
Difficulty	Medium

Time allowed: **50**

Score: **/44**

Percentage: **/100**

Question 1

Given the identity

$$\sin^2\theta + \cos^2\theta \equiv 1$$

prove the following identities:

(i) $\sec^2\theta \equiv 1 + \tan^2\theta$
(ii) $\operatorname{cosec}^2\theta \equiv 1 + \cot^2\theta$

[4 marks]

Question 2

(i) By using the double angle formula for cosine, prove the identity

$$\cos 4\theta \equiv 8\cos^4\theta - 8\cos^2\theta + 1$$

(ii) Show by counter-example that

$$\sin 4\theta \not\equiv 8\sin^4\theta - 8\sin^2\theta + 1$$

[5 marks]

Question 3

(a) Given that θ is small, and that terms involving θ^3 or higher powers of θ can be ignored, use an appropriate approximation to show that

$$4\cos 4\theta - 2\cos^2 2\theta \approx 2 - 24\theta^2$$

[3 marks]

Question 3

(b) Show that the result in part (a) gives a percentage error of 0.583%, to 3 significant figures, when used to approximate

$$4\cos\frac{\pi}{6} - 2\cos^2\frac{\pi}{12}$$

[3 marks]

Question 4

Prove the identity

$$\frac{4\sin^4\theta}{\sin^2 2\theta} \equiv \tan^2\theta \qquad \theta \neq k\pi$$

[3 marks]

Question 5

Show that

$$\sin\theta(\operatorname{cosec}^2\theta - 2) \equiv \frac{\cos 2\theta}{\sin\theta}$$

[4 marks]

Question 6

Use the quotient rule to show that

$$\frac{\mathrm{d}}{\mathrm{d}x}[\tan x] = \sec^2 x$$

[5 marks]

Question 7

Show that

$$\sin 3\theta + \sin\theta \equiv 4\sin\theta - 4\sin^3\theta$$

[5 marks]

Question 8

Prove the identity

$$\frac{4\cot x\cos 2x}{\sin 4x} \equiv \operatorname{cosec}^2 x \qquad x \neq \frac{k\pi}{4}$$

[5 marks]

Question 9

(a) Show that

$$\sin\left(\arccos\left(-\frac{1}{2}\right)\right) = \sqrt{3}\sin\left(\frac{\pi}{6}\right)$$

[2 marks]

Question 9

(b) Show that

$$\arcsin\left(\cos\frac{3\pi}{4}\right) = -\arcsin\left(\cos\frac{\pi}{4}\right)$$

[2 marks]

Question 10

Show that

$$\sqrt{2}\sin\left(\theta - \frac{\pi}{4}\right) \equiv \sin\theta - \cos\theta$$

[3 marks]

3.1 Exponential & Logarithms

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	3. Logs & Exponentials
Topic	3.1 Exponential & Logarithms
Difficulty	Medium

Time allowed: **50**

Score: **/42**

Percentage: **/100**

Question 1

On the same axes, sketch the graphs of $y = 2^x$ and $y = 3^x$, labelling any points where the graphs cross the coordinate axes and writing down the equation of any asymptotes.

[4 marks]

Question 2

(a) Sketch the graph of $y = 2^{-x}$, stating whether this graph indicates exponential growth or exponential decay.

[3 marks]

Question 2

(b) Find the exact value of y when $x = 3$.

[1 mark]

Question 3

Write down the value of a in the following statements.

(i) $\log_a 8 = 3$
(ii) $\log a = 2$
(iii) $\ln e^3 = a$
(iv) $\log_5 a = 1$

[4 marks]

Question 4

(a) Using a calculator, find to 3 significant figures

(i) $\log_2 5 + \log_5 2$
(ii) $\log 25 - \ln 2$
(iii) $\log 200 + \log_5 50 - \log 20 + \ln 10$

[3 marks]

Question 4

(b) Solve $3\log_2 4 + 3x = 5\log_6 216$.

[2 marks]

Question 5

Solve $2^{2x} - 24(2^x) + 128 = 0$.

[3 marks]

Question 6

(a) On the same axes, sketch the graphs of $y = e^x$ and $y = e^{-x}$.
Label any points where the graphs intersect the coordinate axes.
Write down the equations of any asymptotes.

[4 marks]

Question 6

(b) Write down the gradient of $y = e^x$ at the point $(0, 1)$.

[1 mark]

Question 7

(a) Given $y = e^{4x}$ write down an expression for $\frac{dy}{dx}$.

[1 mark]

Question 7

(b) Given $y = 2e^{2x}$ write down an expression for $\frac{dy}{dx}$.

[1 mark]

Question 7

(c) Find the gradient of $y = 3e^{-2x}$ at the point where $x = 3$.
Give your answer in the form pe^q, where p and q are integers to be found.

[3 marks]

Question 8

(a) (i) Write down the gradient of $y = e^{-3x}$.
(ii) Find the gradient of $y = e^{-3x}$ at the point where $x = 0$.

[2 marks]

Question 8

(b) (i) In terms of e, write down the gradient of $y = e^{-3x}$ at the point where $x = 2$.
(ii) Find the value for x for which the gradient of $y = e^{-3x}$ is $-3e^{-12}$.

[2 marks]

Question 9

The function $f(x)$ is defined by $f(x) = 2e^{3x}$ for $x \in \mathbb{R}$.

(a) (i) Find $f(-x)$.
(ii) On the same axes, sketch the graphs of $y = f(x)$ and $y = f(-x)$.
Label any points where the graphs intersect the coordinate axes.

[3 marks]

Question 9

(b) Describe the transformation from $y = f(x)$ to $y = -f(x)$.

[2 marks]

Question 10

Solve $e^{2x} - 8e^{x} + 15 = 0$, giving your answers to 3 significant figures.

[3 marks]

3.2 Modelling with Exponentials & Logarithms

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	3. Logs & Exponentials
Topic	3.2 Modelling with Exponentials & Logarithms
Difficulty	Medium

Time allowed: **70**

Score: **/60**

Percentage: **/100**

Question 1

(a) Write $\left(\frac{1}{3}\right)^x$ in the form e^{kx}.

[1 mark]

Question 1

(b) Write $\left(\frac{2}{7}\right)^t$ in the form e^{kt}.

State whether this would represent exponential growth or exponential decay.

[2 marks]

Question 2

(a) Write $\left(\frac{7}{10}\right)^x$ in the form e^{-kx}.

[1 mark]

Question 2

(b) Sketch the graph of $y = \left(\frac{7}{10}\right)^x$.

State the coordinates of the y-axis intercept.

Write down the equation of the asymptote.

[2 marks]

Question 3

(a) By taking logarithms (base e) of both sides show that the equation

$$y = 5e^{0.1x}$$

can be written as

$$\ln y = 0.1x + \ln 5$$

[1 mark]

Question 3

(b) Given $y = Ae^{kx}$ and $\ln y = 4.1x + \ln 8$, find the values of A and k.

[2 marks]

Question 4

(a) By taking logarithms (base 10) of both sides show that the equation

$$y = 2x^{3.2}$$

can be written as

$$\log y = 3.2 \log x + \log 2$$

[1 mark]

Question 4

(b) Given $y = Ax^b$ and $\log y = 1.8 \log x + \log 5$, find the values of A and b.

[2 marks]

Question 5

(a) By taking logarithms (base 2) of both sides show that the equation

$$y = 3 \times 2^{4x}$$

can be written as

$$\log_2 y = 4x + \log_2 3$$

[1 mark]

Question 5

(b) Given $y = Ab^{kx}$ and $\log_3 y = 5x + \log_3 7$, find the values of A, b and k.

[2 marks]

Question 6

In an effort to prevent extinction scientists released some rare birds into a newly constructed nature reserve.
The population of birds, within the reserve, is modelled by

$$B = 16e^{0.85t}$$

B is the number of birds after t years of being released into the reserve.

(a) Write down the number of birds the scientists released into the nature reserve.

[1 mark]

Question 6

(b) According to this model, how many birds will be in the reserve after 3 years?

[2 marks]

Question 6

(c) How long will it take for the population of birds within the reserve to reach 500?

[2 marks]

Question 7

A manufacturer claims their kettle will keep boiled water hot enough to make a cup of tea for half an hour.
The kettle "boils" water to 90 °C before switching off.
Tea needs to be made with water of a temperature above 77 °C.
A linear model of the temperature, T °C, of the water inside the kettle t minutes after the kettle boils is of the form

$$T = 90 - bt$$

where b is a constant.

(a) Explain the significance of the number 90 in the model.

[1 mark]

Question 7

(b) Assuming the temperature of the water is 77 °C half an hour precisely after the kettle boils, find the value of b.

[2 marks]

Question 7

(c) Find the time at which the temperature has dropped by 2 °C.

[2 marks]

Question 7

(d) A specialist tea website claims that the perfect cup of tea should be made with water at a temperature of no higher than 85 °C.
How many minutes (after the kettle boils) should a user wait before attempting to make the perfect cup of tea?

[1 mark]

Question 7

(e) Explain why the model is redundant for values of t greater than 30.

[1 mark]

Question 8

A simple model for the acceleration of a rocket, A ms^{-2}, is given as

$$A = A_0 e^{0.2t}$$

where t is the time in seconds after lift-off. A_0 is a constant.

(a) What does the constant A_0 represent?

[1 mark]

Question 8

(b) After 10 seconds, the acceleration is 20 ms^{-2}.
Find the value of A_0.

[2 marks]

Question 8

(c) Find how long it takes for the acceleration of the rocket to reach 100 ms^{-2}

[2 marks]

Question 9

Carbon-14 is a radioactive isotope of the element carbon.
Carbon-14 decays exponentially – as it decays it loses mass.
Carbon-14 is used in carbon dating to estimate the age of objects.
The time it takes the mass of carbon-14 to halve (called its half-life) is approximately 5700 years.

A model for the mass of carbon-14, m g, in an object of age t years is

$$m = m_0 e^{-kt}$$

where m_o and k are constants.

(a) For an object initially containing 100g of carbon-14, write down the value of m_0.

[1 mark]

Question 9

(b) Briefly explain why, if $m_0 = 100$, m will equal 50g when $t = 5700$ years.

[2 marks]

Question 9

(c) Using the values from part (b), show that the value of k is 1.22×10^{-4} to three significant figures.

[2 marks]

Question 9

(d) A different object currently contains 60g of carbon-14.
In 2000 years' time how much carbon-14 will remain in the object?

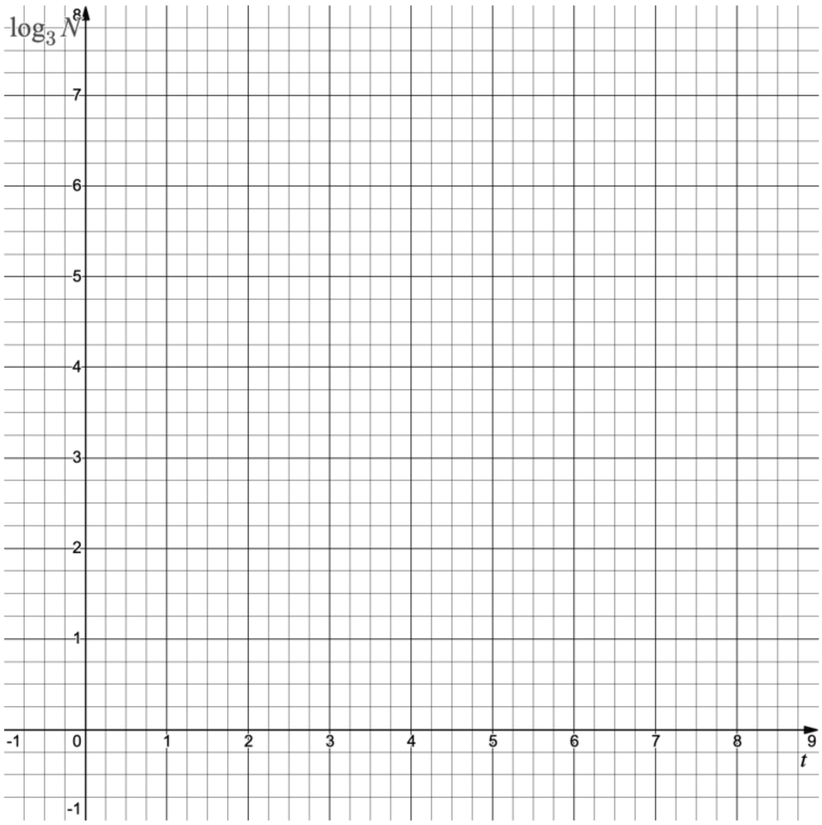
[2 marks]

Question 10

An exponential growth model for the number of bacteria in an experiment is of the form $N = N_0 a^{kt}$. N is the number of bacteria and t is the time in hours since the experiment began. N_0, a and k are constants. A scientist records the number of bacteria at various points over a six-hour period. The results are shown in the table below.

t, hours	0	2	4	6
N, no. of bacteria	100	180	340	620
$\log_3 N$ (3SF)	4.19	4.73	5.31	5.85

(a) Plot the observations on the graph below - plotting $\log_3 N$ against t.



[1 mark]

Question 10

(b) Using the points (0, 4.19) and (6,5.85) find an equation for a line of best fit in the form $\log_3 N = mt + \log_3 c$ where m and c are constants to be found.

[2 marks]

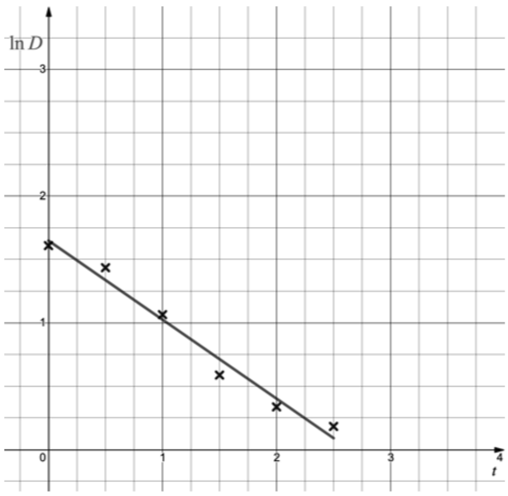
Question 10

(c) The equation $N = N_0 a^{kt}$ can be written in the form $\log_a N = kt + \log_a N_0$.
Use your answer to part (b) to estimate the values of N_0, a, and k.

[2 marks]

Question 11

An exponential model of the form $D = Ae^{-kt}$ is used to model the amount of a pain-relieving drug (D mg/ml) there is in a patient's bloodstream, t hours after the drug was administered by injection. A and k are constants.
The graph below shows values of $\ln D$ plotted against t with a line of best fit drawn.



(a) (i) Use the graph and line of best fit to estimate $\ln D$ at time $t = 0$.

(ii) Work out the gradient of the line of best fit.

[2 marks]

Question 11

(b) Use your answers to part (a) to write down an equation for the line of best fit in the form $\ln D = mt + \ln c$, where m and c are constants.

[1 mark]

Question 11

(c) Show that $D = Ae^{-kt}$ can be rearranged to give $\ln D = -kt + \ln A$

[1 mark]

Question 11

(d) Hence find estimates for the constants A and k.

[2 marks]

Question 11

(e) Find the time when the amount of the pain-relieving drug in the patient's bloodstream is 1.5 mg/ml.

[2 marks]

Question 12

A small company makes a profit of £2500 in its first year of business and £3700 in the second year. The company decides they will use the model

$$P = P_0 y^k$$

to predict future years' profits.
£P is the profit in the y^{th} year of business.
P_0 and k are constants.

(a) Write down two equations connecting P_0 and k.

[2 marks]

Question 12

(b) Find the values of P_0 and k.

[2 marks]

Question 12

(c) Find the predicted profit for years 3 and 4.

[2 marks]

Question 12

(d) Show that

$$P = P_0 y^k$$

can be written in the form

$$\log P = \log P_0 + k \log y.$$

[2 marks]

4.1 Further Differentiation

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	4. Differentiation
Topic	4.1 Further Differentiation
Difficulty	Medium

Time allowed: **60**

Score: **/47**

Percentage: **/100**

Question 1

Given that $\mathrm{f}(x) = \sin x$

(a) Show that

$$\mathrm{f}'(x) = \lim_{h\to 0}\left(\sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)\right)$$

[4 marks]

Question 1

(b) Hence prove that $\mathrm{f}'(x) = \cos x$.

[3 marks]

Question 2

A curve has the equation $y = e^{-3x} + \ln x,\ x > 0$.

Find the gradient of the normal to the curve at the point $(1,\ e^{-3})$, giving your answer correct to 3 decimal places.

[4 marks]

Question 3

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ for each of the following:

(a) $y = \cos(x^2 - 3x + 7) + \sin(e^x)$

[4 marks]

Question 3

(b) $y = \ln(2x^3)$

[3 marks]

Question 4

Find the equation of the tangent to the curve $y = e^{3x^2 + 5x - 2}$ at the point $(-2,\ 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[4 marks]

Question 5

Differentiate with respect to x, simplifying your answers as far as possible:

(a) $(4\cos x - 3\sin x)e^{3x - 5}$

[3 marks]

Question 5

(b) $(x^3 - 4x^2 + 7)\ln x$

[3 marks]

Question 6

Differentiate $\dfrac{5x^7}{\sin 2x}$ with respect to x.

[4 marks]

Question 7

(a) Show that if $y = \operatorname{cosec} 2x$, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -2\operatorname{cosec} 2x \cot 2x$$

[5 marks]

Question 7

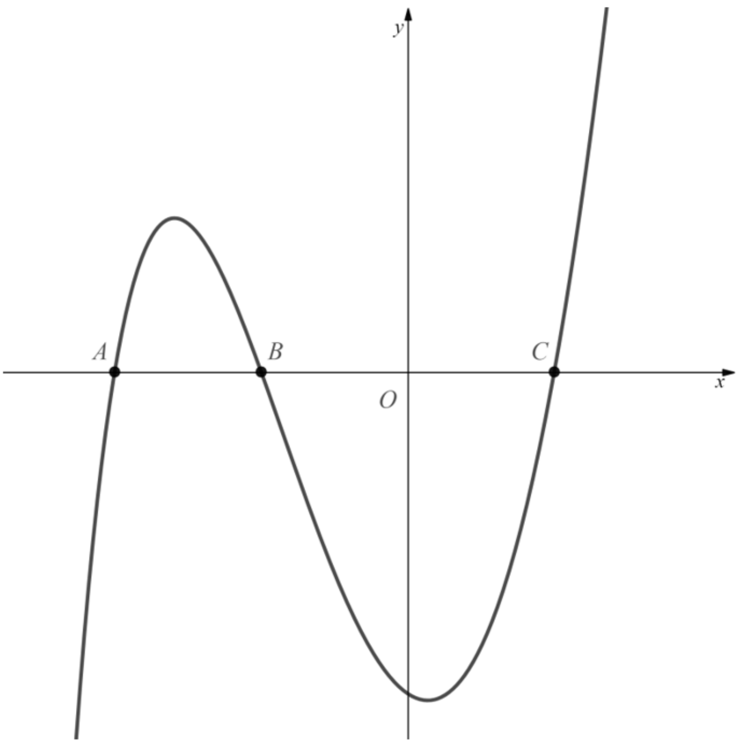
(b) Hence find the gradient of the tangent to the curve $y = \operatorname{cosec} 2x$ at the point with coordinates $\left(\frac{\pi}{3}, \frac{2\sqrt{3}}{3}\right)$.

[1 mark]

Question 8

The diagram below shows part of the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is the function defined by

$$\mathrm{f}(x) = (x^2 - 1)\ln(x + 3), \quad x > 3$$



Points A, B and C are the three places where the graph intercepts the x-axis.

(a) Find $\mathrm{f}'(x)$.

[4 marks]

Question 8

(b) Show that the coordinates of point A are $(-2, 0)$.

[2 marks]

Question 8

(c) Find the equation of the tangent to the curve at point A.

[3 marks]

5.1 Further Integration

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	5. Integration
Topic	5.1 Further Integration
Difficulty	Medium

Time allowed: **70**

Score: **/55**

Percentage: **/100**

Question 1

(a) Find the integral

$$\int \sin x \ \mathrm{d}x$$

[1 mark]

Question 1

(b) Use calculus to evaluate

$$\int_1^4 \frac{1}{x} \, \mathrm{d}x$$

[3 marks]

Question 1

(c) Find an expression for y given that

$$y = \int 7e^{7x} \, \mathrm{d}x$$

[2 marks]

Question 2

(a) Integrate

$$\int \cos 2x \; dx$$

[2 marks]

Question 2

(b) Use calculus to find the value of

$$\int_0^2 (3x-1)^3 \; dx$$

[4 marks]

Question 2

(c) Find an expression for y given that

$$\frac{dy}{dx} = e^{5x}$$

[2 marks]

Question 3

Given that

$$\cos 2\theta \equiv 2\cos^2\theta - 1$$

use calculus to find the exact value of

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2\theta \, \mathrm{d}\theta$$

[6 marks]

Question 4

Find

$$\int \sqrt{1+\cot^2 x} \, \mathrm{d}x$$

[3 marks]

Question 5

(i) Given that $\mathrm{f}(x) = 2x^3 + 4x$, find $\mathrm{f}'(x)$.

(ii) Hence, or otherwise, find

$$\int \frac{3x^2 + 2}{2x^3 + 4x}\,\mathrm{d}x$$

[4 marks]

Question 6

(a) Integrate

$$\int \frac{1}{2x + 3}\,\mathrm{d}x$$

[2 marks]

Question 6

(b) Find an expression for y given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3e^{3x+1}$$

[2 marks]

Question 7

(a) Find an expression for $\mathrm{f}(x)$ given that

$$\mathrm{f}(x) = \int \sin(2x+1)\ \mathrm{d}x$$

[2 marks]

Question 7

(b) Find an expression for y given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3\cos(3x+5)$$

[2 marks]

Question 7

(c) Use calculus to find the exact value of

$$\int_{\frac{7\pi}{6}}^{\frac{4\pi}{3}} \sec^2(x - \pi)\ \mathrm{d}x$$

[3 marks]

Question 8

(a) Use the identity

$$\sin 2A \equiv 2 \sin A \cos A$$

to show that

$$4 \sin\frac{\theta}{2} \cos\frac{\theta}{2} \equiv 2 \sin\theta$$

[2 marks]

Question 8

(b) Hence, or otherwise, find the integral

$$\int 4\sin\frac{\theta}{2}\cos\frac{\theta}{2}\ \mathrm{d}\theta$$

[3 marks]

Question 9

Show that

$$\int_2^5 4e^{2x-4}\ \mathrm{d}x = 2(e^6 - 1)$$

[3 marks]

Question 10

Given that the graph of $y = \mathrm{f}(x)$ passes through the point $(0, 4)$ and that

$$\mathrm{f}'(x) = \frac{12x^2 + 7}{2(4x^3 + 7x + 4)}$$

(a) Find $\mathrm{f}(x)$

[4 marks]

Question 10

(b) Explain why $x = -\frac{1}{2}$ must be excluded from the domain of $\mathrm{f}(x)$.

[1 mark]

Question 11

Show that

$$\int \tan x \ \mathrm{d}x = \ln|\sec x| + c$$

where c is a constant.

[4 marks]

6.1 Numerical Methods

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	6. Numerical Methods
Topic	6.1 Numerical Methods
Difficulty	Medium

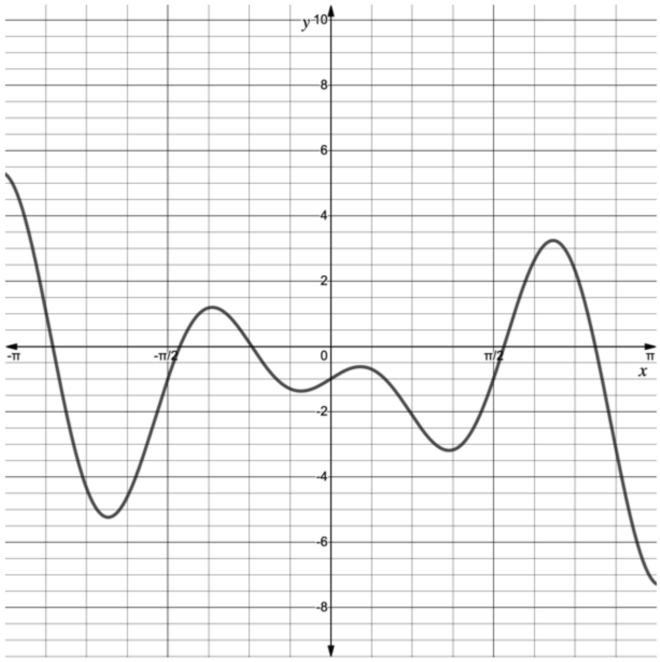
Time allowed: **50**

Score: **/38**

Percentage: **/100**

Question 1

The diagram below shows part of the graph $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = 2x\cos(3x) - 1$.



(a) (i) Find f(1.6) and f(1.7), giving your answers to three significant figures.

(ii) Briefly explain the significance of your results from part (i).

[3 marks]

Question 1

(b) One of the solutions to the equation $\mathrm{f}(x) = 0$ is $x = 2.55$, correct to three significant figures.

(i) Write down the upper and lower bound of 2.55.

(ii) Hence, use the sign change rule to confirm that this is a solution (to three significant figures) to the equation $\mathrm{f}(x) = 0$.

[3 marks]

Question 2

(a) Show that the equation $x^3 + 3 = 5x$ can be rewritten as

$$x = \sqrt[3]{5x - 3}$$

[2 marks]

Question 2

(b) Starting with $x_0 = 1.8$, use the iterative formula

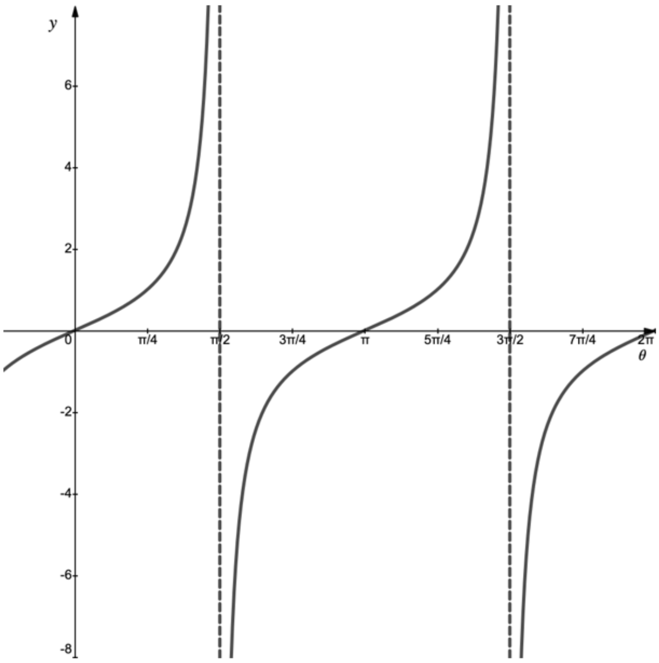
$$x_{n+1} = \sqrt[3]{5x_n - 3}$$

to find a root of the equation $x^3 + 3 = 5x$, correct to two decimal places.

[3 marks]

Question 3

Part of the graph of $y = \tan\theta$ is shown below, where θ is measured in radians.

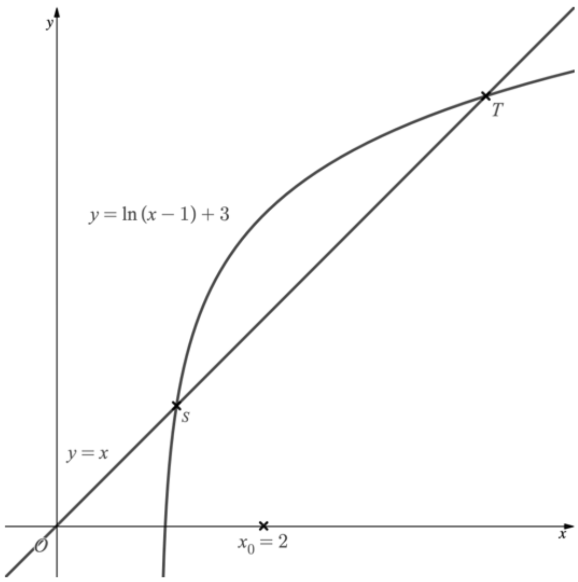


Explain why the change of sign rule would fail if attempting to locate a root of the function $f(\theta) = \tan\theta$ using the values of $\theta = 1.55$ and $\theta = 1.65$.

[2 marks]

Question 4

The diagram below shows the graphs of $y = x$ and $y = \ln(x - 1) + 3$.



The iterative formula

$$x_{n+1} = \ln(x_n - 1) + 3$$

is to be used to find an estimate for a root, α, of the function $f(x)$.

(a) Write down an expression for $f(x)$.

[1 mark]

Question 4

(b) Using an initial estimate, $x_0 = 2$, show, by adding to the diagram above, which of the two points (S or T) the sequence of estimates $x_1, x_2, x_3, \ldots$ will converge to.
Hence deduce whether α is the x-coordinate of point S or point T.

[2 marks]

Question 4

(c) Find the estimates x_1, x_2, x_3 and x_4, giving each to three decimal places.

[2 marks]

Question 4

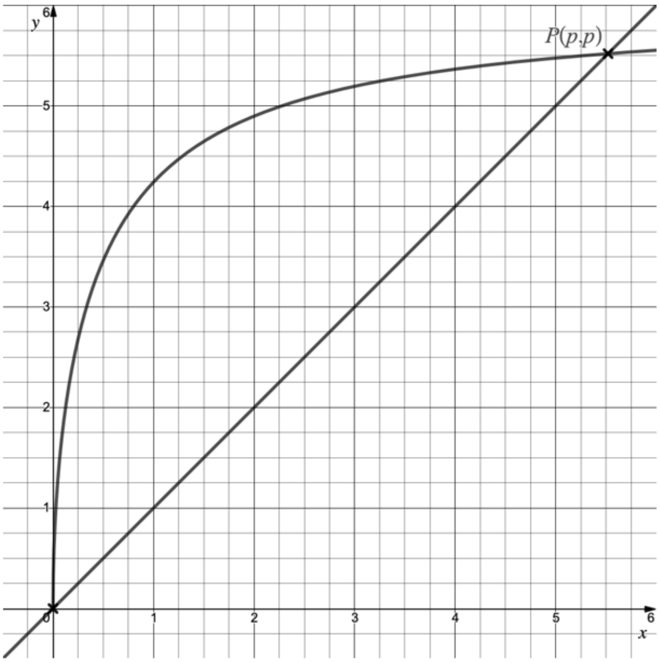
(d) Confirm that $\alpha = 4.146$ correct to three decimal places.

[2 marks]

Question 5

The village of Greendale lies on a straight road, as modelled by the line $y = x$ on the graph below. To ease rush hour congestion, a bypass is to be built around Greendale.

The path of the bypass is modelled by the equation $y = 6\sqrt{1 - \frac{1}{x+1}}$.



The bypass runs from the origin to the point $P(p, p)$.

(a) On the diagram show how using the iterative formula $x_{n+1} = 6\sqrt{1 - \frac{1}{x_n+1}}$ with $0 < x_0 < p$ will lead to convergence at the point P.

[1 mark]

Question 5

(b) Use the iterative method $x_{n+1} = 6\sqrt{1 - \frac{1}{x_n+1}}$ with $x_0 = 5$ to find the value of p, correct to two decimal places.

[3 marks]

Question 5

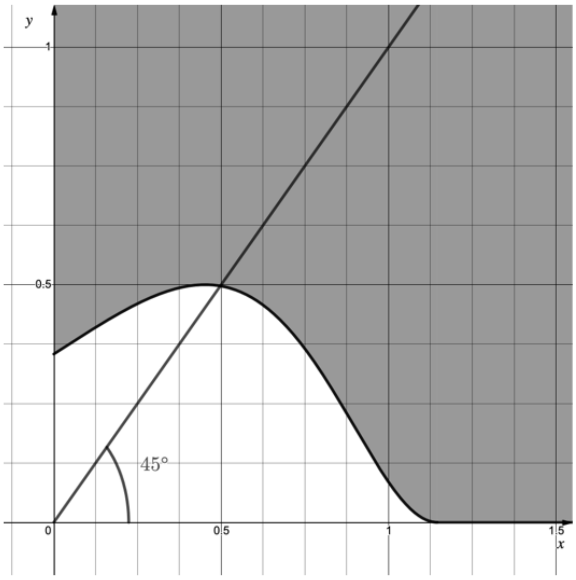
(c) Use the sign change rule with the function $f(x) = x - 6\sqrt{1 - \frac{1}{x+1}}$ to show your answer to part (b) is correct to two decimal places.

[2 marks]

Question 6

The game of Curveball is played on a flat table.
A player rolls a ball from a fixed point, at any angle, with the aim of it coming to rest in the winning zone.

A particular player decides to roll the ball at an angle of 45°.
This is illustrated by the graph below with the ball being rolled from the origin and the shaded area being the winning zone.



The boundary of the winning zone is given by part of the curve with equation

$$y = \frac{1}{2}\sin^2(e^x).$$

(a) Use the iterative formula $x_{n+1} = \frac{1}{2}\sin^2(e^x)$ with initial starting value $x_0 = 0.5$, to show that the x-coordinate of the point where this player's ball should cross the winning zone boundary is 0.497 to three significant figures.

[3 marks]

Question 6

(b) Use your answer to part (a) to find the minimum distance the ball should travel for this player to win Curveball.

[2 marks]

Question 7

According to legend, a unicorn can heal an injury almost instantly by touching it with its horn.

When a unicorn touches a cut in human skin of length L mm, it will heal according to the model

$$\mathrm{f}(t) = Le^{-t} - t \qquad\qquad t \geq 0$$

where f is the length of the cut in millimetres, at time t seconds after the unicorn has touched the injury with its horn.

(a) (i) Write down the value f would be when the cut is completely healed.

(ii) Show that, for a cut in human skin of length 5 mm the equation $\mathrm{f}(t) = 0$ can be rearranged into the form

$$t = -\ln\left(\frac{t}{5}\right).$$

[2 marks]

Question 7

(b) Use the iterative formula $t_{n+1} = -\ln\left(\frac{t_n}{5}\right)$, with initial value $t_0 = 1$, to find how many seconds it takes a cut of size 5 mm to heal once a unicorn has touched it with its horn.

(i) Write down the values of the estimates t_1, t_2 and t_3 to four decimal places.
(ii) Give your final answer to two significant figures.
(iii) State the number of iterations required for convergence to two significant figures.

[4 marks]

Question 7

(c) Briefly explain why the model should also restrict the range of $\mathrm{f}(t)$ to be greater than or equal to zero?

[1 mark]