1.2 Quadratics

Question Paper

Course	Edexcel IAL Maths: Pure 1
Section	1. Algebra & Functions
Topic	1.2 Quadratics
Difficulty	V. Hard

Time allowed: **70**

Score: **/57**

Percentage: **/100**

Question 1

(a) Write the quadratic function $y = -6x^2 + 8x - 5$ in the form $y = a - b(x + c)^2$ where a, b and c are constants to be found.

[2 marks]

Question 1

(b) Write down the maximum point on the graph of $y = -6x^2 + 8x - 5$.

[1 mark]

Question 1

(c) Sketch the graph of $y = -6x^2 + 8x - 5$, clearly labelling the maximum point and any point where the graph intersects the coordinate axes.

[3 marks]

Question 2

(a) The equation $y = x^2 + px + q$ and no real roots. Show that $p^2 < 4q$ and explain why q must be a positive value.

[2 marks]

Question 2

(b) Given that the minimum point on the graph of $y = x^2 + px + q$ is $(3, 1)$ find the values of p and q.

[2 marks]

Question 3

The equation $k^2x^2 - 4x + 5 = k^2$ has two distinct real roots.
Find the possible values of k.

[2 marks]

Question 4

(a) The equation $4k - 6kx - x^2 = 0$ has two distinct real roots, α and β.
k is a negative constant and $0 < \alpha < \beta$.
Sketch the graph of $y = 4k - 6kx - x^2$, labelling the points where the graph crosses the coordinate axes.

[2 marks]

Question 4

(b) Find the possible values of k.

[3 marks]

Question 5

(a) Find the minimum value of the function $f(x) = x^2 + 8x + c$, giving your answer in terms of c.

[2 marks]

Question 5

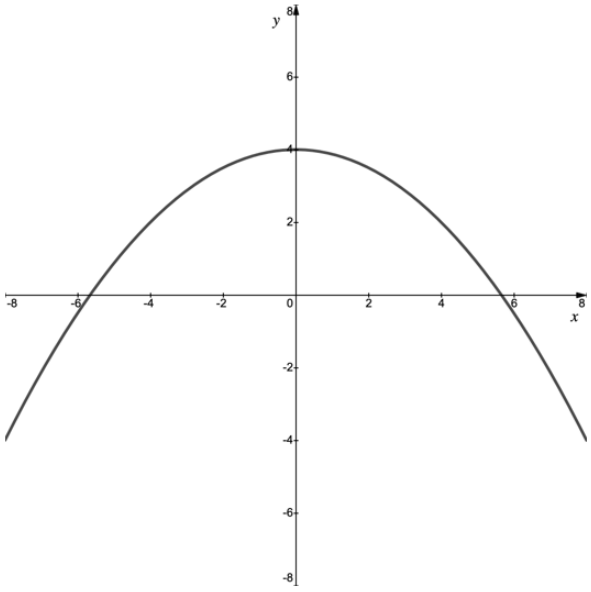
(b) Find the values of c for which the function $f(x) = x^2 + 8x + c$ has no real roots.

[2 marks]

Question 6

The graph below shows the curve $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = 4 - \frac{x^2}{8}$.

The curve is to be used as the model for the arch on a bridge where the water level under the bridge is represented by the x-axis. All measurements are in meters.



(a) Depending on rainfall throughout the year, the water level can rise by up to 0.5 m, determine whether the bridge is still wide enough to span a river of width 11 m when it is at its peak height.

[2 marks]

Question 6

(b) A barge in the shape of a cuboid (above water level) has a cross-section measuring 6 m wide by 2.5 m tall. The barge regularly travels along the river where the bridge is to be built. Justifying your answer, determine if the barge will fit underneath the bridge or not.

[2 marks]

Question 6

(c) To support the bridge the arch will continue 2.5 m under the water (ground) level. Find the exact distance between the base of the arch on either side of the river.

[2 marks]

Question 7

(a) Show that the equation $ax^2 + bx + c = 0$ can be written in the form $a(x + \frac{b}{2a})^2 - \frac{b^2-4ac}{4a} = 0$

[2 marks]

Question 7

(b) Hence show that $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

[2 marks]

Question 8

The function f(x) is defined by f$(x) = (k-1)x^2 - (k-2)x - 2k,\ x \in \mathbb{R}$.
The function g(x) is defined by g$(x) = (k-1)x^2 - 3kx + k + 1,\ x \in \mathbb{R}$.
k is a non-zero constant and $k \neq 1$.

(a) The graphs of $y = \text{f}(x)$ and $y = \text{g}(x)$ intersect once.
Find the x-coordinate of the intersection, giving your answer in terms of k.

[3 marks]

Question 8

(b) In the case when $k = 3$, find the coordinates of the point of intersection of $y = \text{f}(x)$ and $y = \text{g}(x)$.

[2 marks]

Question 9

(a) Solve the equation $8\sqrt{x} = 48 - x$.

[3 marks]

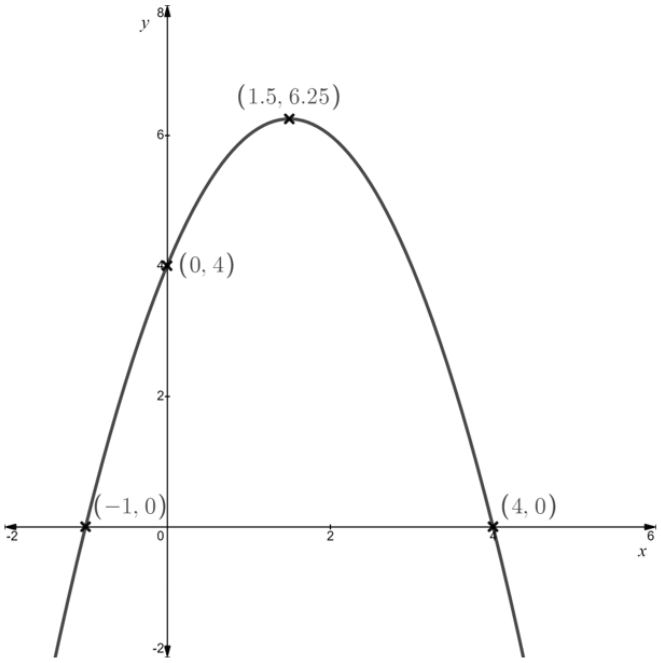
Question 9

(b) Solve the equation $2^{4x} + 64 = 20(2^{2x})$.

[3 marks]

Question 10

The diagram below shows the graph of $y = f(x)$. The intercepts with the coordinate axes and the turning point have been labelled.



The graph is transformed by the function $y = f(x) + 6$. One of the new x-axis intercepts is (-2, 0).

Sketch the graph of $y = f(x) + 6$, stating the coordinates of any points that intersect the coordinate axes and the turning point.

[3 marks]

Question 11

A stone is thrown vertically upwards from the top of a cliff. The path of the stone is modelled by the quadratic function $h(t) = 52 + 3t - 0.5t^2$, $t \geq 0$, where h is the height, in meters, of the stone above the sea and t is the time in seconds since the stone was thrown.

(a) Write down the height of the cliff from which the stone was thrown.

[1 mark]

Question 11

(b) Find the maximum height the stone reaches above the sea.

[2 marks]

Question 11

(c) How long does it take for the stone to hit the sea?

[2 marks]

Question 11

(d) How long does the stone stay above it's starting height for?

[2 marks]

Question 12

(a) Factorise $x^2 + 6x + 9$

[1 mark]

Question 12

(b) Factorise $x^2 + 6xy + 9y^2$

[2 marks]

Question 12

(c) Find a relationship between x and y such that $x^2 + 6xy + 9y^2 = 0$

[2 marks]