4.1 Differentiation

Question Paper

Course	Edexcel IAL Maths: Pure 1
Section	4. Differentiation
Topic	4.1 Differentiation
Difficulty	V. Hard

Time allowed: 70

Score: /57

Percentage: /100

Question 1

For each of the following, find $\dfrac{dy}{dx}$ in terms of x:

(a) $y = -\dfrac{5}{4}x^3 + \dfrac{3}{5}x^2 - x\sqrt{2} + \pi$

[2 marks]

Question 1

(b) $y = \dfrac{3}{2}x^{\frac{4}{5}} - \dfrac{10}{3}x^{-\frac{4}{5}}$

[2 marks]

Question 2

Given that $y = \left(\dfrac{1}{x} - \dfrac{1}{x\sqrt{x}} \right)^2$, $x > 0$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

[4 marks]

Question 3

For each of the following, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x:

(a) $y = \dfrac{2x^3 - 5x^2 - 3x}{2x + 1}$

[3 marks]

Question 3

(b) $y = \left(\sqrt{x} + 3 - \dfrac{4}{\sqrt{x}}\right)^2$

[4 marks]

Question 4

The function f is defined by $f(x) = 2x^3 + px^2 + 3x - 16.$ Determine the range of values for p for which the equation $f'(x) = 0$ has at least one real solution.

[5 marks]

Question 5

A curve has the equation $y = x\sqrt{x} + \dfrac{48}{\sqrt{x}}$, $x > 0$. Find the coordinates of the point on the curve where the gradient is 0.

[5 marks]

Question 6

The function f is defined by $f(x) = x^n - x$, $n \in \mathbb{N}, n \geq 2$. Determine the relationship between the value of n and the number of real solutions to the equation $f'(x) = 0$.

[4 marks]

Question 7

A curve is described by the equation $\dfrac{\sqrt{y}}{-1+\sqrt{x}} = \dfrac{1}{x}$, $x > 1$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

[3 marks]

Question 8

The curve with equation $y = ax^2 + bx + c$ passes through the point $(-1, 4)$. At the point $(2, 7)$ the gradient of the curve is 7. Find the values of a, b and c.

[5 marks]

Question 9

A curve has equation $y = 5 - (x - 3)^2$.

A is the point on the curve with x coordinate 0, and B is the point on the curve with x coordinate 6.

C is the point of intersection of the tangents to the curve at A and B.

(a) Find the coordinates of point C.

[7 marks]

Question 9

(b) Calculate the area of triangle ABC.

[2 marks]

Question 10

A curve is described by the equation $y = f(x)$, where

$$f(x) = \frac{1}{\sqrt{x}}, \quad x > 0$$

P is the point on the curve such that the normal to the curve at P also passes through the origin.

(a) Find the coordinates of point P. Give your answer in the form $(2^a, 2^b)$, where a and b are rational numbers to be found.

[6 marks]

Question 10

(b) Write down the equation of the normal to the curve at P.

[1 mark]

Question 10

(c) Show that an equation of the tangent to the curve at P is

$$\left(2^{\frac{1}{3}}\right) x + \left(2^{\frac{5}{6}}\right) y = 3$$

[4 marks]