

# 4.5 Modelling with Sequences & Series

## Question Paper

Course	Edexcel IAL Maths: Pure 2
Section	4. Sequences & Series
Topic	4.5 Modelling with Sequences & Series
Difficulty	Hard

**Time allowed:** 50

**Score:** /42

**Percentage:** /100

**Question 1**

June is to start training in order to run a marathon.

For the first week of training she will run a total of 3 miles.

Each subsequent week she'll increase the total number of miles run by  $x$  miles.

June intends training for  $y$  weeks and will run a total distance of 570 miles during the training period.

- (a) Write down an expression in  $x$  for the number of miles June will run in the 10<sup>th</sup> week of training.

**[2 marks]**

**Question 1**

- (b) Write down an equation in  $x$  and  $y$  for the total distance June will run during the training period.

**[2 marks]**

**Question 1**

- (c) Given that June runs twice as far in week 4 than in week 2, find the values of  $x$  and  $y$ .

**[2 marks]**

**Question 2**

Stephen opens a savings account with £600.

Compound interest is paid annually at a rate of 1.2%.

At the start of each new year Stephen pays another £600 into his account.

(a) Show that at the end of two years Stephen has £1221.69 in the account

**[2 marks]**

**Question 2**

(b) Show that at the end of year  $n$ , the amount of money, in pounds, Stephen will have in his account is given by

$$600(1.012 + 1.012^2 + 1.012^3 + \dots + 1.012^n)$$

**[2 marks]**

**Question 2**

(c) Hence show that the total amount, in pounds, in Stephen's account after  $n$  years is

$$50\,600(1.012^n - 1).$$

**[2 marks]**

### Question 2

(d) Find the year in which the amount in Stephen's account first exceeds £10 000.

**[2 marks]**

### Question 2

(e) State one assumption that has been made about this scenario.

**[1 mark]**

### Question 3

A ball is dropped and bounces such that the height of each bounce is 80% of the previous bounce.

The first bounce of the ball reaches a height of 1.60 m.

(a) Find the height the ball bounces on its 8<sup>th</sup> bounce.

**[2 marks]**

### Question 3

(b) The ball is considered as no longer bouncing once its bounce height fails to reach 1% of its first bounce height.

Find the number of bounces the ball makes.

**[2 marks]**

**Question 3**

(c) Find the total distance travelled by the ball from when it first hits the ground to when it stops bouncing.

**[2 marks]**

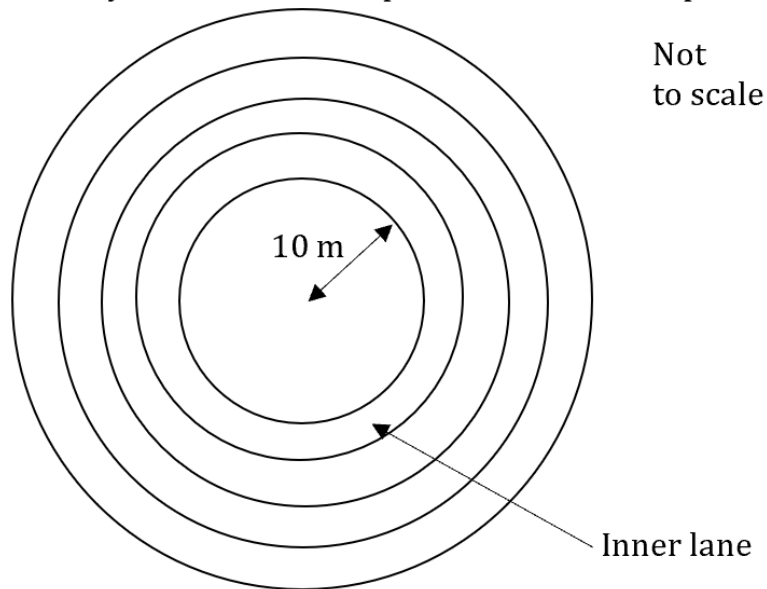
**Question 3**

(d) Give one reason why this model may be unrealistic.

**[1 mark]**

**Question 4**

A training track for cyclists is in the shape of a circle made up of several lanes.



The shortest, inner lane has a radius of 10 m, with each lane working outwards having a radius 4 m greater than the previous lane.

During a training session a cyclist is expected to complete two laps of each lane, starting with the inner lane, before moving onto the next one.

You may assume that the lap distance of each lane is the circumference of the circle with the radius indicated above.

(a) Show that the lap distances of each lane form an increasing arithmetic sequence and state the first term and common difference.

**[3 marks]**

**Question 4**

(b) Find the distance completed by a cyclist during a training session and using the first six lanes only.

**[2 marks]****Question 4**

There is a total of 10 lanes on the training track.

If a cyclist wishes to travel a total distance more than twice round each lane, they may continue as many laps around the outside lane as necessary.

(c) A more advanced cyclist wants to travel a distance of at least 6 km during their session. How many laps of the outside lane the cyclist will need to do in total?

**[3 marks]****Question 5**

A healthy unicorn will breathe out one million particles of magic dust with every breath. However, as a unicorn dies the particles of magic dust in each breath reduces by 20%. A unicorn's last breath is the first to contain under 100 particles of magic dust.

(a) If a unicorn breathes out every 10 seconds find the time it takes for the unicorn to die from the moment it takes its last healthy breath.

**[3 marks]**

### Question 5

(b) Including its last healthy breath, find the total number of magic dust particles a unicorn breathes out as it dies.

**[2 marks]**

### Question 5

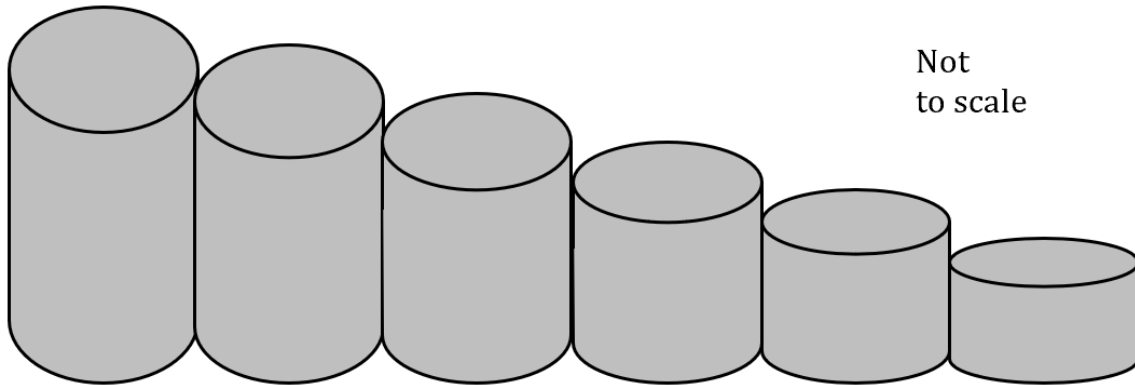
(c) Show, that even if it continued to live and breathe with under 100 particles of magic dust in each breath, a dying unicorn will never breath out more than 5 000 000 particles of magic dust in total.

**[2 marks]**



**Question 6**

A model maker is constructing part of a model building using a series of hollow tubes, stacked next to each other as illustrated below.



Each tube is one-tenth shorter than the one to its left.

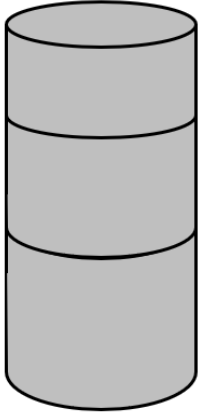
All tubes are the same width as they are all cut from one longer tube.

- (a) Show that no matter how many tubes the model maker uses, the longer tube they are cut from need not be any longer than ten times the height of the tallest tube.

**[2 marks]**

### Question 6

In a different part of the model building, tubes are required to be stacked on top of each other, as shown below.



Not  
to scale

The tallest tube is the same length as the tallest tube above.

All the other tubes are 1 cm shorter than the tube immediately below it.

The longer tube all others are cut from is ten times the height of the tallest tube in the stack.

All of the longer tube shall be used in the stack.

(b) If the length of the tallest tube is  $a$  cm and there are  $n$  tubes in the stack, show that  $n^2 - (2a + 1)n + 20a = 0$ .

**[3 marks]**