1.2 Functions

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	1. Algebra & Functions
Topic	1.2 Functions
Difficulty	Hard

Time allowed: **70**

Score: **/55**

Percentage: **/100**

Question 1

State whether the following mappings are one-to-one, many-to-one, one-to-many or many-to-many.

(i) $\mathrm{f}: x \mapsto 2 - x^3$

(ii) $\mathrm{f}: x \mapsto \sin x$

(iii) $\mathrm{f}: x \mapsto \frac{1}{x^2}$

(iv) $\mathrm{f}: x \mapsto \ln x$

[4 marks]

Question 2

It is given

$$\mathrm{f}(x) = \frac{2}{x}$$

(a) Write down the domain of the function $\mathrm{f}(x)$.

[1 mark]

Question 2

(b) Sketch the graph of $y = \mathrm{f}(x)$, stating the coordinates of any intersections with the coordinate axes and the equations of any asymptotes.

[3 marks]

Question 2

(c) Write down the range of $f(x)$.

[1 mark]

Question 3

The function $f(x)$ is defined as

$$f(x) = x(x+3)^2 + 1 \qquad x \geq 0$$

(a) Work out the range of $f(x)$.

[1 mark]

Question 3

(b) If the domain of $f(x)$ is changed to $x \leq 0$, what is the range of $f(x)$?

[2 marks]

Question 4

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = 3x^2 + 2 \qquad x \in \mathbb{R}$$
$$g(x) = 1 - 3x \qquad x \in \mathbb{R}$$

(a) Write down the range of $f(x)$.

[1 mark]

Question 4

(b) Find (i) $fg(x)$
(ii) $gf(x)$

[4 marks]

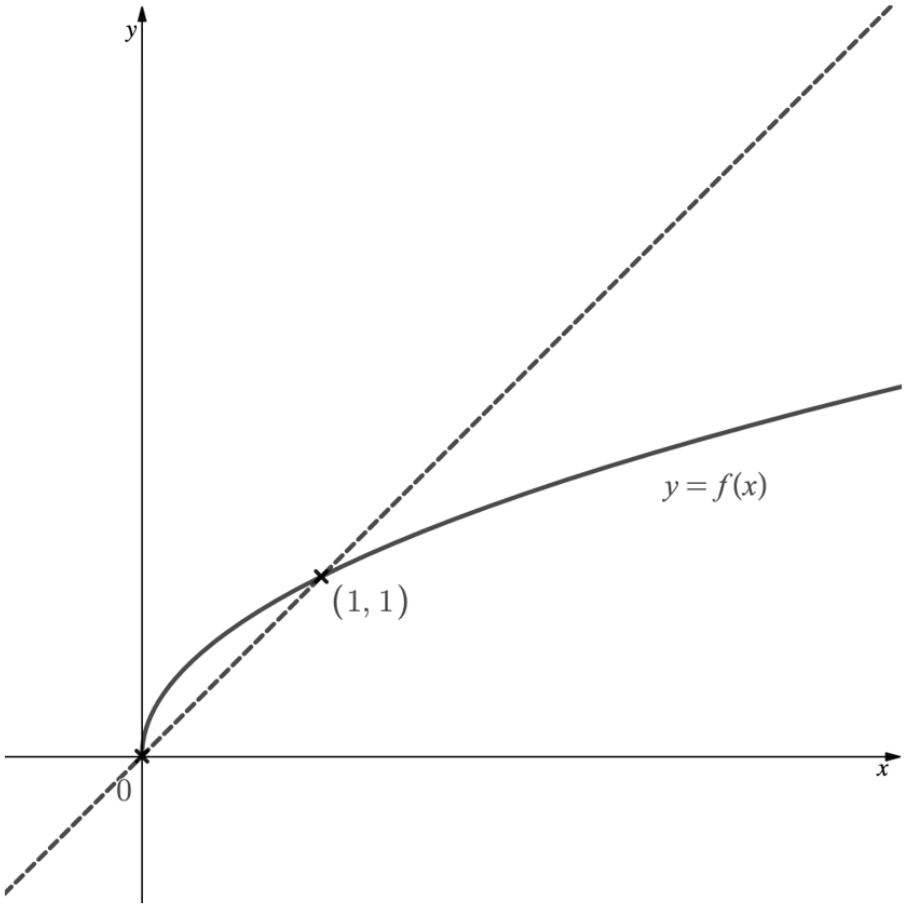
Question 4

(c) Solve the equation $f(x) = g(x) + 1$

[2 marks]

Question 5

The graph of $y = f(x)$ is shown below.



(a) (i) Use the graph to write down the domain and range of $f(x)$.

(ii) Write down the equation of the dotted line on the graph.

[3 marks]

Question 5

(b) On the diagram above sketch the graph of $y = \mathrm{f}^{-1}(x)$.

[2 marks]

Question 6

(a) On the same axes, sketch the graphs of $y = |\mathrm{f}(x)|$ and $y = |\mathrm{g}(x)|$ where

$$\mathrm{f}(x) = 3x - 1 \qquad x \in \mathbb{R}$$
$$\mathrm{g}(x) = 2x + 2 \qquad x \in \mathbb{R}$$

Label the points at which the graphs intersect the coordinate axes.

[3 marks]

Question 6

(b) Solve the equation $|\mathrm{f}(x)| = |\mathrm{g}(x)|$.

[3 marks]

Question 6

(c) Which of the solutions to $|f(x)| = |g(x)|$ is also a solution to $f(x) = g(x)$?

[1 mark]

Question 7

The function $f(x)$ is defined as

$$f: x \mapsto |3x - 2| \qquad x \in \mathbb{R}$$

(a) Explain why the inverse of $f(x)$ does not exist.

[1 mark]

Question 7

(b) Suggest an adaption to the domain of $f(x)$ so its inverse does exist, but also produces the maximum possible range for $f(x)$.

[1 mark]

Question 7

(c) Using your adaption from part (b), find an expression for $f^{-1}(x)$ and state its domain and range.

[3 marks]

Question 8

Solve the equation $|x^2 - 4| = 3$, giving your answers in exact form.

[3 marks]

Question 9

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = e^{x-2} \qquad x \in \mathbb{R}$$
$$g(x) = 2 + \ln x \qquad x \in \mathbb{R}, x > 0$$

(a) Find (i) $fg(x)$
(ii) $gf(x)$

[3 marks]

Question 9

(b) Write down $f^{-1}(x)$ and state its domain and range.

[2 marks]

Question 9

(c) The graphs of $f(x)$ and $f^{-1}(x)$ are drawn on the same axes.
Describe the transformation that would map one graph onto the other.

[2 marks]

Question 10

The functions $f(x)$, $g(x)$ are defined as follows

$$f(x) = |x - 2| - 5 \qquad x \in \mathbb{R}$$
$$g(x) = |x| \qquad x \in \mathbb{R}$$

(a) Sketch the graph of $y = gf(x)$, stating the coordinates of all points where the graph intercepts the coordinate axes.

[4 marks]

Question 10

(b) (i) How many solutions are there to the equation $\text{gf}(x) = 1$?

(ii) How many solutions are there to the equation $\text{gf}(x) = 10$?

[2 marks]

Question 10

(c) Solve the equation $\text{gf}(x) = 2$.

[3 marks]