1.3 Combinations of Transformations

Question Paper
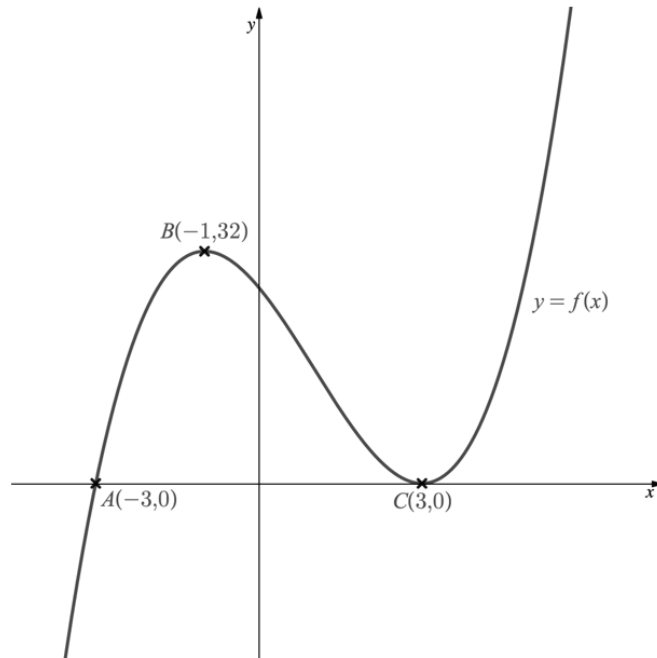
Course	Edexcel IAL Maths: Pure 3
Section	1. Algebra & Functions
Topic	1.3 Combinations of Transformations
Difficulty	Hard

Time allowed: 60

Score: /51

Percentage: /100

Question 1

The diagram below shows the graph of $y = f(x)$.
The stationary points and intercepts with the x-axis are marked on the diagram.

On separate diagrams, sketch the graphs with equations
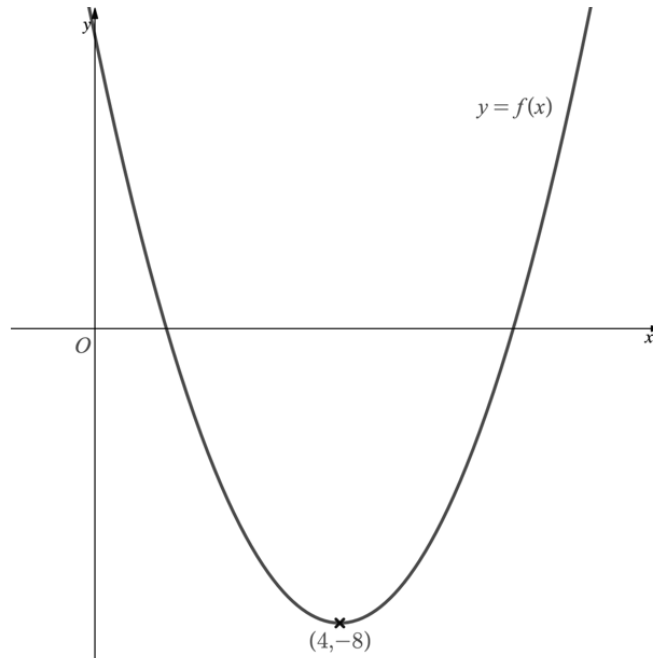
(i) $y = f\left(\frac{1}{2}x\right) + 2,$

(ii) $y = -f(x - 1).$

On each diagram, mark the coordinates of the images of the points A, B and C under the given transformation.

[6 marks]

Question 2

The minimum point on the graph of $y = f(x)$ has coordinates $(4, -8)$ as shown on the diagram below.

(a) Sketch the graph of $y = |f(2x)| - 3$ and state the coordinates of the maximum point.

[3 marks]

Question 2

(b) Find the exact distance between the minimum point on the graph of $y = f(x)$ and the maximum point on the graph of $y = |f(2x)| - 3$.

[2 marks]

Question 3

Describe, in order, a sequence of transformations that maps the graph of $y = f(x)$ onto the following graphs:

(i) $y = -f(3x - 1)$,
(ii) $y = 2f(5 - x)$.

[4 marks]

Question 4

Given that $f(x) = \ln(2x + 1)$ find an expression for $g(x)$, where $g(x)$ is obtained by applying the following sequence of transformations to $f(x)$.

1. Translation by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$,
2. Horizontal stretch by scale factor $\frac{1}{2}$,
3. Reflection in the x-axis.

[4 marks]

Question 5

(a) On the same axes sketch the graphs of $y = \text{p}(x)$ and $y = p^{-1}(x)$, where $\text{p}(x) = |2x|, \ x \leq 0.$

[3 marks]

Question 5

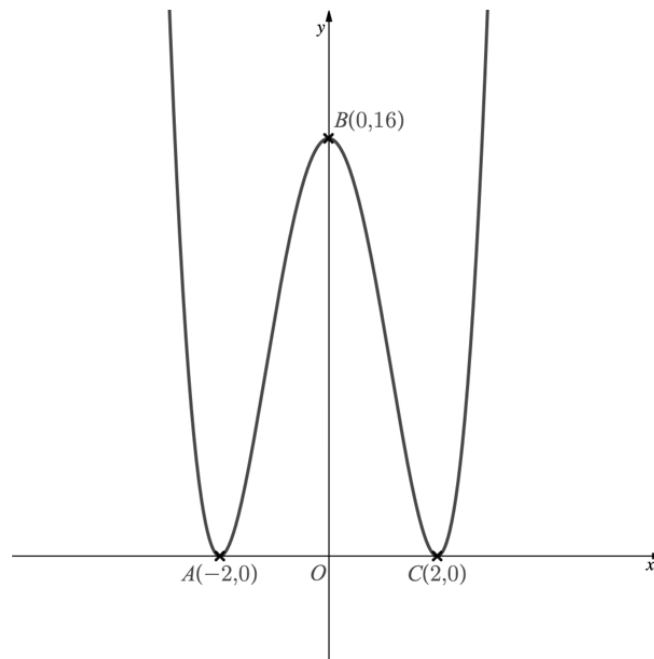(b) Find an expression for $\text{p}^{-1}(x)$ and state its domain.

[3 marks]

Question 5

(c) Show that $p^{-1}(x) = -\frac{1}{2}p(-\frac{1}{2}x)$.

[3 marks]

Question 6

A sketch of the graph with equation $y = f(x)$, where $f(x) = (x^2 - 4)^2$ is shown below.

The points A, B and C are the points where the graph intercepts the coordinate axes.

(a) Sketch the graph of $y = -3f(2x)$, labelling the images of the three points A, B and C.
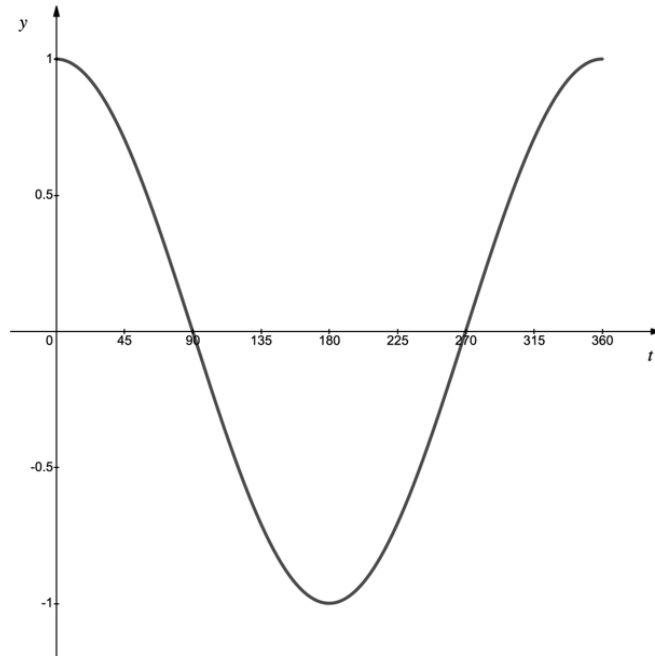
[3 marks]

Question 6

(b) Suggest a combination of at least two transformations that will transform the points A, B and C such that none of them lie on the coordinate axes.

Give your answer in the form of an expression in terms of f(x).

[2 marks]

Question 7

The diagram shows the graph of $y = f(t)$, where $f(t) = \cos t, \ 0° \le x \le 360°$.

(a) (i) Write down the maximum value of y when $y = -2f(3t)$.

(ii) Write down the value of t for which this maximum occurs.

[2 marks]

Question 7

(b) Find, in terms of $f(t)$, the combination of transformations that would map the graph of $y = f(t)$ onto the graph of $y = 2 - 4\sin t, \ 0° \le x \le 180°$.

[2 marks]

Question 8

The function $f(x)$ is to be transformed by a sequence of functions, in the order detailed below.

1. A translation by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$
2. A reflection in the y-axis
3. A vertical stretch by scale factor $\frac{2}{3}$
4. A translation by $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

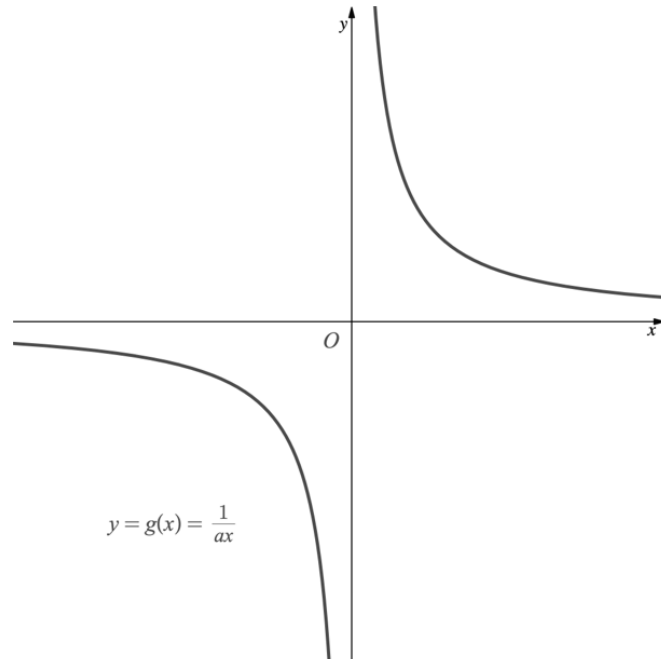Write down the combined transformation in terms of $f(x)$.

[3 marks]

Question 9

The diagram below shows the graph of $y = g(x)$ where

$$g(x) = \frac{1}{ax}, \qquad a, x \neq 0$$

where a is a constant.

$$y = g(x) = \frac{1}{ax}$$

(a) (i) Write down the equations of the asymptotes on the graph of $y = g(x)$.

(ii) Determine the equations of the asymptotes on the graph of $y = 3g(2x + 1)$.

[5 marks]

Question 9

(b) Determine the domain and range of the series of transformations to $y = f(x)$ where

$$f(x) = -2g\left(\frac{1}{3}x + 3\right) - 4.$$

[3 marks]

Question 10

The point with coordinates $(-3, -5)$ is a stationary point on the graph with equation $y = h(x)$.

Determine the coordinates of the stationary point on graphs with the following equations:

(i) $y = |h(2x) - 5|$,

(ii) $y = h\left(\frac{1}{4}x + 1\right)$,

(iii) $y = 2 - \frac{1}{5}h\left(\frac{1}{2}x\right)$.

[3 marks]