2.4 Trigonometric Proof

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	2. Trigonometry
Topic	2.4 Trigonometric Proof
Difficulty	Hard

Time allowed: **60**

Score: **/50**

Percentage: **/100**

Question 1

Given the identity

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

prove the following identities:

(i) $\cos 2\theta \equiv \cos^2 \theta - \sin^2 \theta$
(ii) $\cos 2\theta \equiv 1 - 2\sin^2 \theta$
(iii) $\cos 2\theta \equiv 2\cos^2 \theta - 1$

[4 marks]

Question 2

(i) Prove the identity

$$\sin 3\theta \equiv 3\sin \theta - 4\sin^3 \theta$$

(ii) Show by counter-example that

$$\cos 3\theta \not\equiv 3\cos \theta - 4\cos^3 \theta$$

[5 marks]

Question 3

(a) Given that θ is small, and that terms involving θ^3 or higher powers of θ can be ignored, show that

$$\frac{1}{\text{cosec}^2\left(\frac{\theta}{2}\right)} + \frac{1}{\sec^2\left(\frac{\theta}{4}\right)} \approx 1 + \frac{3}{16}\theta^2$$

[3 marks]

Question 3

(b) Determine the percentage error when the result in part (a) is used to approximate

$$\frac{1}{\text{cosec}^2\left(\frac{7}{20}\right)} + \frac{1}{\sec^2\left(\frac{7}{40}\right)}$$

giving your answer correct to 3 significant figures.

[3 marks]

Question 4

Prove that

$$\cot^2\theta - \tan^2\theta \equiv 4\cot 2\theta \operatorname{cosec} 2\theta$$

[5 marks]

Question 5

Show that

$$\cos 4\theta + \cos\frac{\pi}{3} \equiv 8\sin^4\theta - 8\sin^2\theta + \frac{3}{2}$$

[5 marks]

Question 6

Prove the identity

$$\frac{1-\tan^2 x}{\cos 2x} \equiv \sec^2 x \qquad x \neq \frac{2k+1}{4}\pi$$

[5 marks]

Question 7

Prove the identity

$$\operatorname{cosec} x \equiv \frac{\frac{1}{2}\sec^2 \frac{x}{2}}{\tan \frac{x}{2}}$$

[4 marks]

Question 8

Show that

$$\tan\frac{x}{2} \equiv \frac{1}{\operatorname{cosec} x + \cot x} \qquad x \neq 2k\pi$$

[5 marks]

Question 9

(a) Given that $y = \arcsin(kx)$, where k is a constant, show that $x = \frac{1}{k}\cos\left(\frac{\pi}{2} - y\right)$.

[3 marks]

Question 9

(b) Hence show that the value of $\arcsin kx + \arccos kx$ is constant and independent of k. Find the value of this constant.

[3 marks]

Question 10

Show that

$$\frac{10}{4\cos\theta + 3\sin\theta} \equiv 2\sec(\theta - \alpha)$$

where

$$\alpha = \arctan\left(\frac{3}{4}\right)$$

[5 marks]