3.2 Modelling with Exponentials & Logarithms

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	3. Logs & Exponentials
Topic	3.2 Modelling with Exponentials & Logarithms
Difficulty	Hard

Time allowed: **70**

Score: **/60**

Percentage: **/100**

Question 1

(a) Write $\left(\frac{3}{5}\right)^x$ in the form e^{kx}, giving the value of k to three significant figures.

[2 marks]

Question 1

(b) Write $\left(\frac{4}{7}\right)^{3t}$ in the form e^{kt}, giving the value of k to three significant figures.

State, and justify, whether this would represent exponential growth or decay.

[2 marks]

Question 2

(a) Write $(0.7)^{x+1}$ in the form Ae^{-kx}.

[2 marks]

Question 2

(b) Sketch the graph of $y = (0.7)^{x+1} - 3$.

State the coordinates of the y-axis intercept.

Write down the equation of the asymptote.

[2 marks]

Question 3

(a) Show that the equation

$$x = 7e^{-0.2t}$$

can be written as

$$\ln x = \ln 7 - 0.2t$$

[2 marks]

Question 3

(b) Rewrite the equation $\ln y = 4.1x + \ln 8$ in the form $y = Ae^{kx}$.

[2 marks]

Question 4

(a) Show that the equation

$$y = 2x^{\frac{3}{4}}$$

can be written as

$$\log y = 0.75 \log x + \log 2$$

[2 marks]

Question 4

(b) Rewrite the equation $\log y = 4.7 \log x + \log 12$ in the form $y = Ax^b$.

[2 marks]

Question 5

(a) Show that the equation

$$y = 0.1 \times 2^{0.01x}$$

can be written as

$$\log_2 y = 0.01x - \log_2 10$$

[2 marks]

Question 5

(b) Rewrite the equation $\log_3 y = 6.3x + \log_3 4$ in the form $y = Ab^{kx}$.

[2 marks]

Question 6

Scientists introduced a small number of rare breed deer to a large wildlife sanctuary.
The population of deer, within the sanctuary, is modelled by

$$D = 20e^{0.1t}$$

D is the number of deer after t years of first being introduced to the sanctuary.

(a) Write down the number of deer the scientists introduced to the sanctuary.

[1 mark]

Question 6

(b) How many years does it take for the deer's population to double?

[2 marks]

Question 6

(c) Give one criticism of the model for population growth.

[1 mark]

Question 6

(d) The scientists suggest that the population of deer are separated after either 25 years or when their population exceeds 400.
Find the earliest time the deer should be separated.

[2 marks]

Question 7

A manufacturer claims their flask will keep a hot drink warm for up to 8 hours.
In this sense, warm is considered to be 40°C or higher.
It is assumed a hot drink has an initial temperature of 80°C.
A linear model of the temperature, T °C, inside the flask t hours from when a hot drink is first made is of the form

$$T = a + bt$$

where a and b are constants.

(a) Write down the value of a.

[1 mark]

Question 7

(b) Assuming that a hot drink has a temperature of 40°C after 8 hours, find the value of b.

[2 marks]

Question 7

(c) When does the model predict the temperature has decreased by 20°C.

[1 mark]

Question 7

(d) Suggest a problem if the model were to be used for values of t larger than 8.

[1 mark]

Question 8

A simple model for the acceleration of a rocket, A ms^{-2}, is given as

$$A = 5e^{kt}$$

where t is the time in seconds after lift-off. k is a constant.

(a) After 4 seconds the acceleration of the rocket is 10 ms^{-2}.
Find the value of k.

[2 marks]

Question 8

(b) Find the time at which the acceleration of the rocket has increased by 200%.

[2 marks]

Question 8

(c) Sketch the graph of the acceleration of the rocket, against time, stating the coordinates of the point that shows the initial acceleration of the rocket.

[2 marks]

Question 9

Carbon-14 is a radioactive isotope of the element carbon.
Carbon-14 decays exponentially – as it decays it loses mass.
Carbon-14 is used in carbon dating to estimate the age of objects.
The time it takes the mass of carbon-14 to halve (called its half-life) is approximately 5700 years

A model for the mass of carbon-14, y g, in an object originally containing 100 g, at time t years is

$$y = 100e^{-kt}$$

where k is a constant.

(a) Find the value of k, giving your answer to three significant figures.

[2 marks]

Question 9

(b) The object is considered as having no radioactivity once the mass of carbon-14 it contains falls below 0.5 g. Find out how old the object would have to be considered non-radioactive.

[2 marks]

Question 9

(c) A different object currently contains 25g of carbon-14.
In 500 years' time how much carbon-14 will remain in the object?

[2 marks]

Question 10

An exponential growth model for the number of bacteria in an experiment is of the form

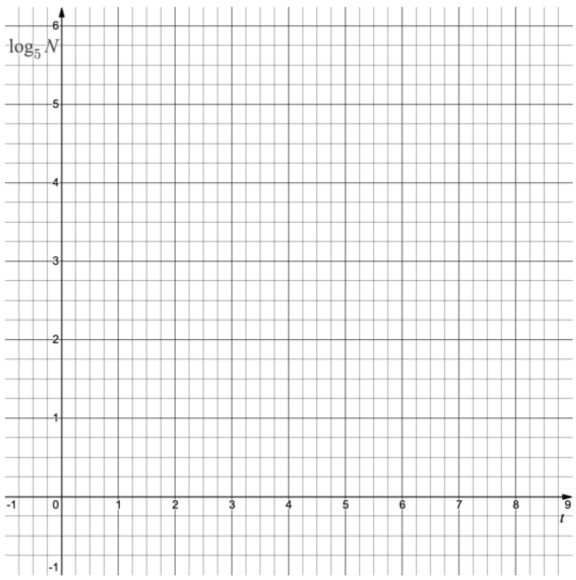
$$N = N_0 a^{kt}.$$

N is the number of bacteria and t is the time in hours since the experiment began.
N_0, a and k are constants.

A scientist records the number of bacteria at various points over a six-hour period.
The results are in the table below.

t, hours	0	2	4	6
N, no. of bacteria	200	350	600	1100

(a) Plot the observations on the graph below - plotting $\log_5 N$ against t.
Draw a line of best fit.



[2 marks]

Question 10

(b) Find an equation for your line of best fit in the form $\log_5 N = mt + \log_5 c$.

[2 marks]

Question 10

(c) Estimate the values of N_0, a, and k.

[2 marks]

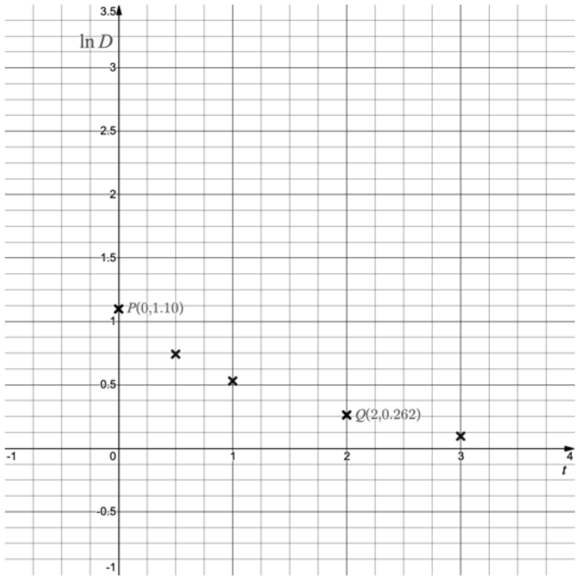
Question 11

An exponential model of the form

$$D = Ae^{-kt}$$

is used to model the amount of a pain-relieving drug (D mg/ml) there is in a patient's bloodstream, t hours after the drug was administered by injection. A and k are constants.

The graph below shows values of $\ln D$ plotted against t.



(a) Using the points marked P and Q, find an equation for the line of best fit, giving your answer in the form $\ln D = mt + \ln c$, where m and c are constants to be found.

[2 marks]

Question 11

(b) Hence find estimates for the constants A and k.

[2 marks]

Question 11

(c) The patient is allowed a second injection of the drug once the amount of drug in the bloodstream falls below 1% of the initial dose.
Find, to the nearest minute, how long until the patient is allowed a second injection of the drug.

[2 marks]

Question 12

The annual profits, in thousands of pounds, of a small company in the first 4 years of business are given in the table below.

a, years in business	1	2	3	4
P, annual profit	£3100	£4384	£5369	£6200

Using this data the company uses the model

$$P = P_1 a^k$$

to predict future years' profits. P_1 and k are constants.

(a) Use data from the table to find the values of P_1 and k.

[2 marks]

Question 12

(b) Show that $\log P = k \log a + \log P_1$, where P_1 and k take the values found in part (a).

[2 marks]

Question 12

(c) State a potential problem with using the model to predict the profit in the company's 12^{th} year of business.

[1 mark]