3.2 Modelling with Exponentials & Logarithms

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	3. Logs & Exponentials
Topic	3.2 Modelling with Exponentials & Logarithms
Difficulty	Easy

Time allowed: **40**

Score: **/36**

Percentage: **/100**

Question 1

State whether the following functions could represent exponential growth or exponential decay.

(i) $f(x) = 5e^{2x}$

(ii) $f(t) = 100e^{-t}$

(iii) $f(a) = 20e^{-ka},\ k > 0$

(iv) $f(t) = Ae^{kt},\ A, k > 0$

[4 marks]

Question 2

Write the following in the form e^{kx}, where k is a constant and $k > 0$.

(i) $e^{3x} \times e^{2x}$

(ii) 5^x

(iii) 2^x

[3 marks]

Question 3

Write the following in the form e^{-kx}, where k is a constant and $k > 0$.

(i) $\dfrac{e^{-2x}}{e^{4x}}$

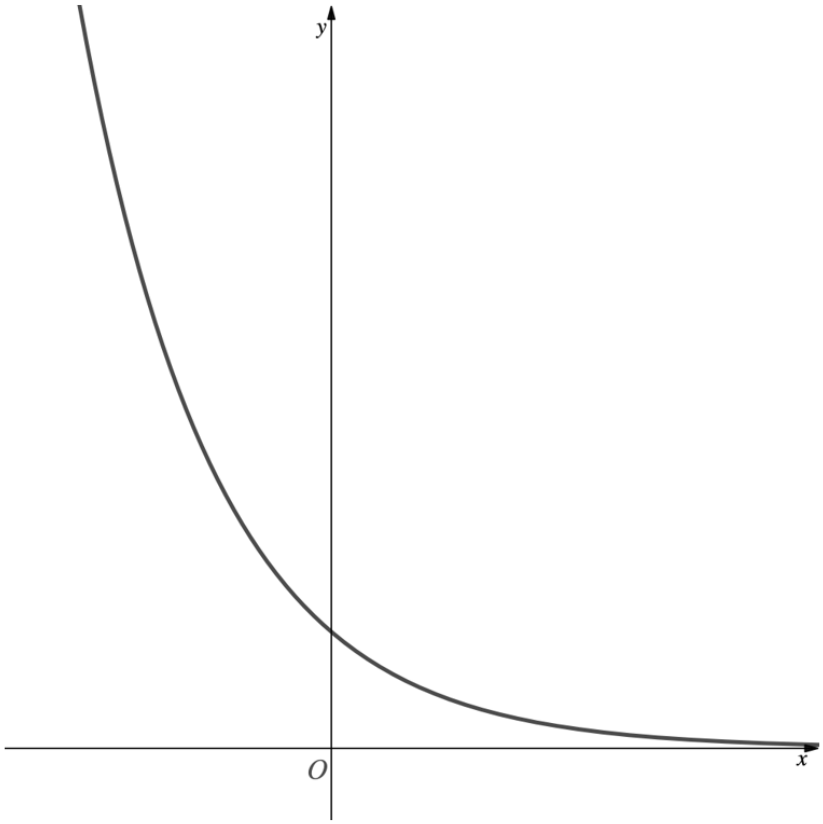
(ii) $\left(\dfrac{1}{5}\right)^{x}$

(iii) $\left(\dfrac{1}{2}\right)^{x}$

[3 marks]

Question 4

The diagram below shows a sketch of the graph of $y = e^{-x}$.
On the diagram, add the graph of $y = e^{-2x}$ labelling the point at which the graph intersects the y-axis.
Write down the equation of any asymptotes on the graph.



[3 marks]

Question 5

(a) By taking logarithms (base e) of both sides show that the equation

$$y = Ae^{kx}$$

can be written in the form $\ln y = kx + \ln A$

[3 marks]

Question 5

(b) Hence ...

(i) ... write the equation $y = 2e^{0.01x}$ in the form $\ln y = kx + \ln A$.

(ii) ... write the equation $\ln y = 0.3x + \ln 5$ in the form $y = Ae^{kx}$.

[4 marks]

Question 6

In an effort to prevent extinction scientists released 24 rare birds into a newly constructed nature reserve.
The population of birds, within the reserve, is modelled by

$$B = Ae^{0.4t}$$

B is the number of birds after t years of being released into the reserve.
A is a constant.

(a) Write down the value of A.

[1 mark]

Question 6

(b) According to this model, how many birds will be in the reserve after 2 years?

[2 marks]

Question 6

(c) How many years after release will it take for the population of birds to double?

[2 marks]

Question 7

A simple model for the acceleration of a rocket, A ms^{-2}, is given as

$$A = 10e^{0.1t}$$

where t is the time in seconds after lift-off.

(a) What is the meaning of the value 10 in the model?

[1 mark]

Question 7

(b) Find the acceleration of the rocket 15 seconds after lift-off.

[2 marks]

Question 7

(c) Find how long it takes for the acceleration to reach 100 ms^{-2}.

[3 marks]

Question 8

An exponential growth model for the number of bacteria in an experiment is of the form $N = Ae^{kt}$.

N is the number of bacteria and t is the time in hours since the experiment began.

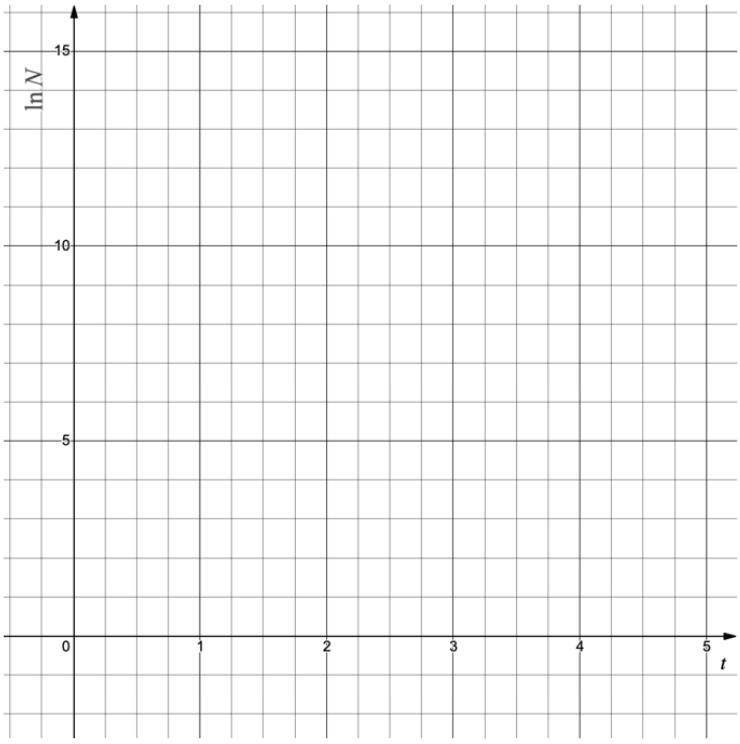
A and k are constants.

A scientist records the number of bacteria every hour for 3 hours.

The results are shown in the table below.

t, hours	0	1	2	3	4
N, no. of bacteria	100	210	320	730	1580
$\ln N$ (3SF)	4.61	5.35	5.77	6.59	7.37

(a) Plot the observations on the graph below - plotting $\ln N$ against t.



[1 mark]

Question 8

(b) Using the points (0, 4.61) and (4, 7.37), find an equation for a line of best fit in the form $\ln N = mt + \ln c$, where m and c are constants to be found.

[2 marks]

Question 8

(c) Hence estimate the values of A and k.

[2 marks]