4.1 Further Differentiation

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	4. Differentiation
Topic	4.1 Further Differentiation
Difficulty	Easy

Time allowed: 50

Score: /44

Percentage: /100

Question 1

Given that $f(x) = x^2$

(a) Use differentiation from first principles to show that

$$f'(x) = \lim_{h \to 0} \left(\frac{(x^2 + 2hx + h^2 - x^2)}{h} \right).$$

[2 marks]

Question 1

(b) Hence prove that

$$f'(x) = 2x.$$

[3 marks]

Question 2

A curve has the equation $y = 5e^{-2x}$.

(a) Find an expression for $\frac{dy}{dx}$.

[2 marks]

Question 2

(b) (i) Find the gradient of the tangent at the point where $x = 1$, giving your answer in the form $-ae^{-2}$ where a is a positive integer to be found.

(ii) Hence show that the gradient of the normal to the curve at the point where $x = 1$ is $\dfrac{1}{10}e^2$.

[3 marks]

Question 3

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ for

(i) $y = \sin(3x^2),$
(ii) $y = 2\ln(x^3).$

[4 marks]

Question 4

The curve with equation $y = e^{x^2 - 9}$ passes through the point with coordinates $(-3, 1)$.

(i)   Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(ii)   Find the equation of the tangent to the curve at the point $(-3, 1)$.

[4 marks]

Question 5

(a) Differentiate $(x^3 - 2x) \ln x$ with respect to x.

[3 marks]

Question 5

(b) Differentiate $e^x \cos 2x$ with respect to x.

[3 marks]

Question 6

(a) Differentiate $\dfrac{\cos x}{\sin x}$ with respect to x.

[3 marks]

Question 6

(b) Differentiate $\dfrac{2x^2-3x+4}{\sin 3x}$ with respect to x.

[3 marks]

Question 7

Write down $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when

    (i) $y = \sec 5x$,

    (ii) $y = \operatorname{cosec} 3x$.

[2 marks]

Question 8

The function $f(x)$ is defined as

$$f(x) = (x^2 - 4x + 4)\ln(x), \qquad x > 0$$

(a) Show that the graph of $y = f(x)$ intercepts the x-axis at the points $(1, 0)$ and $(2, 0)$.

[4 marks]

Question 8

(b) Find $f'(x)$.

[4 marks]

Question 8

(c) Find the gradient of the tangent at the point $(1, 0)$.

[2 marks]

Question 8

(d) Hence find the equation of the tangent at the point $(1, 0)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

[2 marks]