6.1 Numerical Methods

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	6. Numerical Methods
Topic	6.1 Numerical Methods
Difficulty	Hard

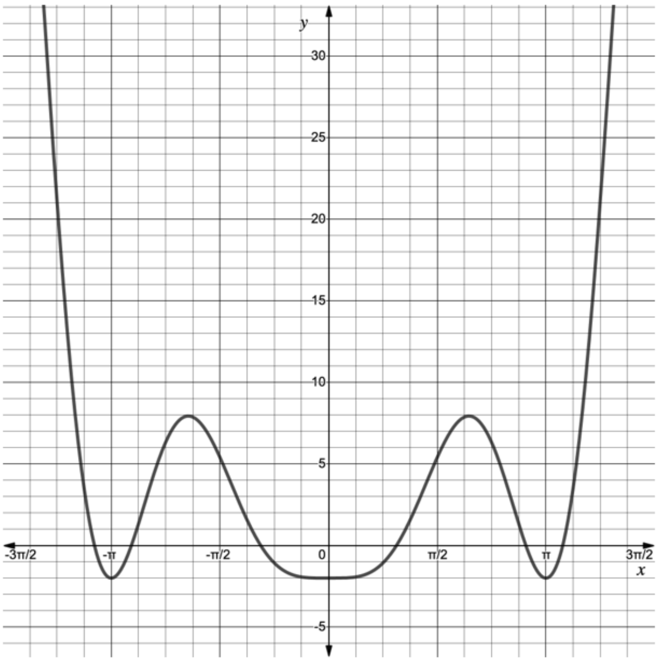
Time allowed: **50**

Score: **/43**

Percentage: **/100**

Question 1

The diagram below shows part of the function $y = f(x)$ where $f(x) = 3x^2 \sin^2 x - 2$.



(a) Correct to three significant figures, $f(0.9) = -0.509$ and $f(3.4) = 0.265$.

Explain why using the sign change rule with these values would not necessarily be helpful in finding the root close to $x = 0.98$.

[2 marks]

Question 1

(b) Using suitable values of x, show that there is a root close to $x = 0.98$.

[2 marks]

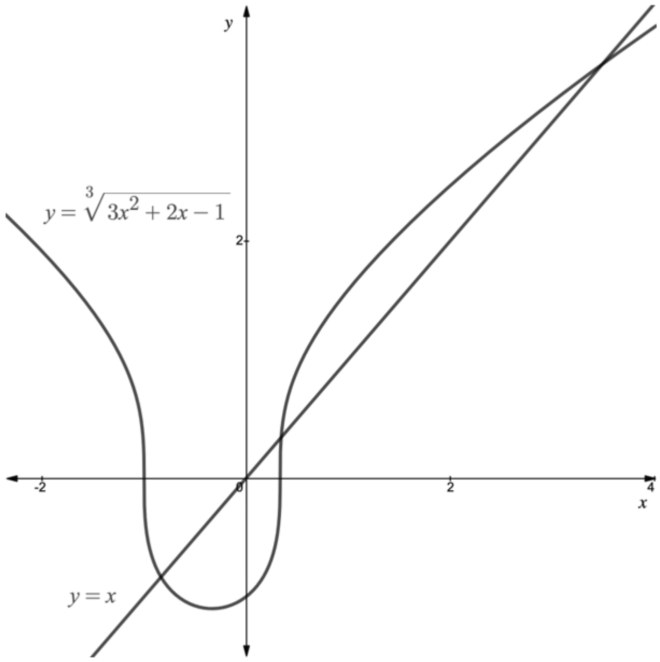
Question 1

(c) Show that the root close to $x = 0.98$ is 0.982, correct to three significant figures.

[2 marks]

Question 2

The diagram below shows a sketch of the graphs $y = x$, and $y = \sqrt[3]{3x^2 + 2x - 1}$.



(a) An iterative formula is used to find roots to the equation $x^3 - 3x^2 - 2x + 1 = 0$. On the diagram above show that the iterative formula

$$x_{n+1} = \sqrt[3]{3x_n^{\ 2} + 2x_n - 1}$$

would converge to the root close to $x = 3.5$ when using a starting value of $x_0 = 0.5$.

[2 marks]

Question 2

(b) (i) Use $x_0 = 0.5$ in the iterative formula from part (a) to find three further approximations to the root close to $x = 3.5$.
Give each approximation correct to three significant figures.

(ii) Comment on your approximations and what they suggest about convergence to the root close to $x = 3.5$.

[2 marks]

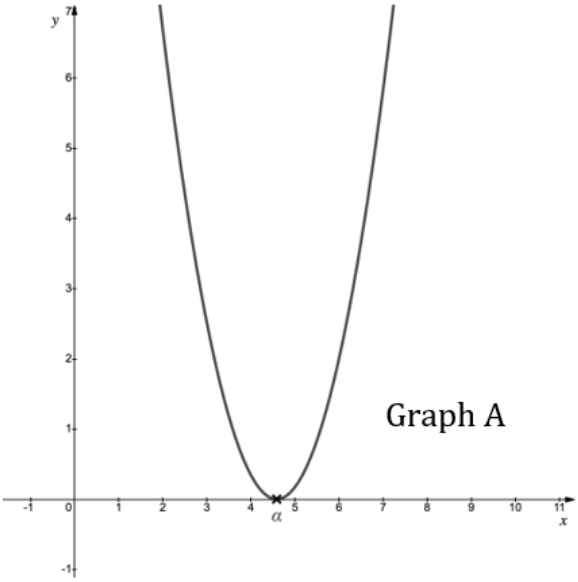
Question 2

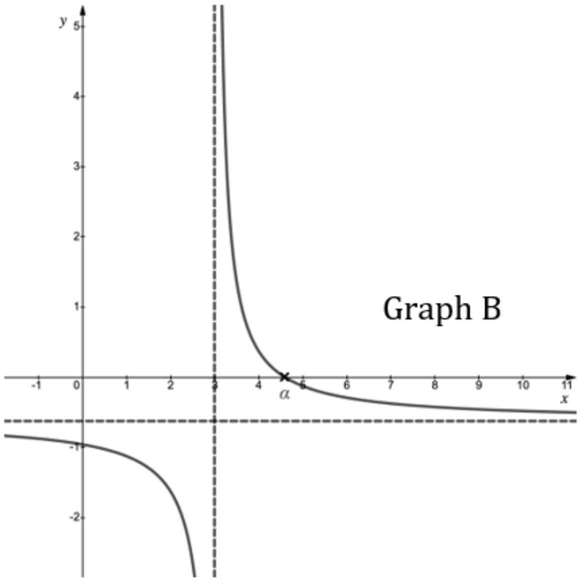
(c) Confirm that the root close to $x = 3.5$ is 3.49 correct to three significant figures.

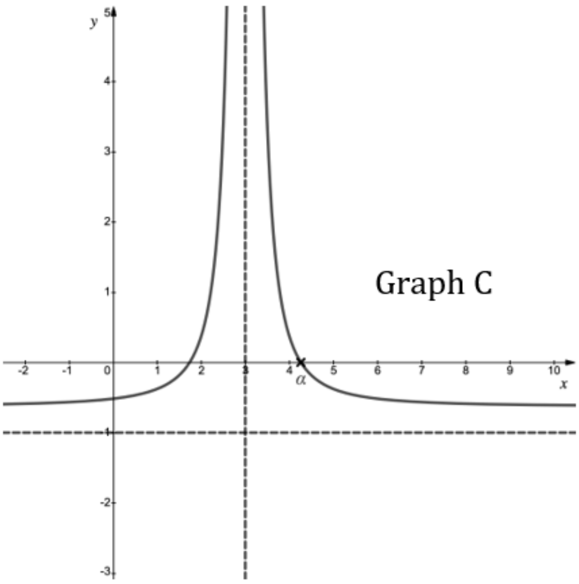
[3 marks]

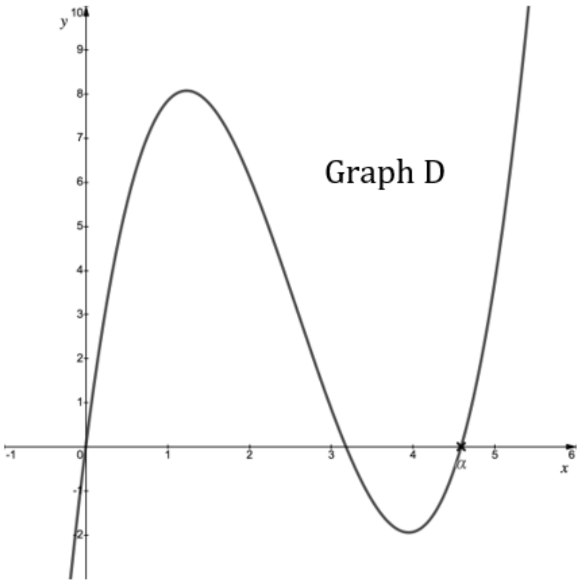
Question 3

The diagrams below show the graphs of four different functions.









Match each graph above with the correct statement below.

(1) The sign change rule with values of $x = 2$ and $x = 4$ would indicate a root but has failed due to the discontinuity (asymptote) at $x = 3$.

(2) The sign change rule with values of $x = 1$ and $x = 5$ would indicate no root but has failed because there are two roots in the interval $(1, 5)$.

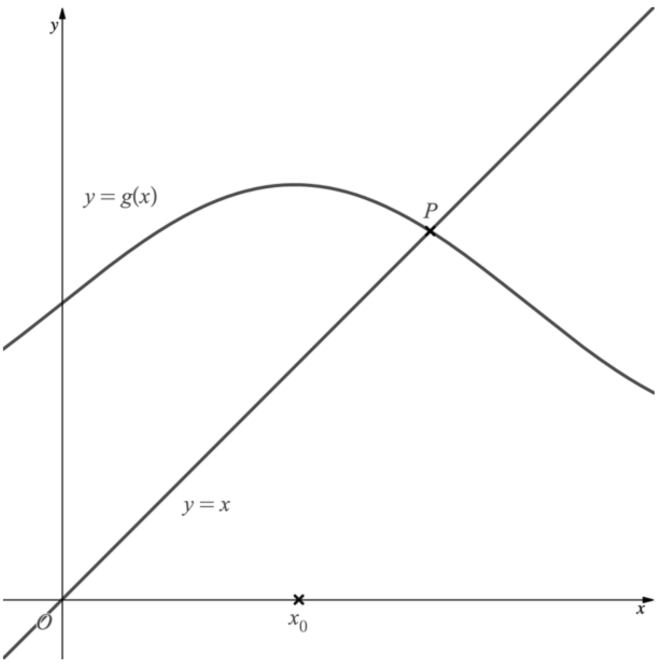
(3) The sign change rule with values of $x = 3$ and $x = 5$ would indicate no root but fail as there are two roots in the interval $(3, 5)$.

(4) The sign change rule with values of $x = 3$ and $x = 5$ would indicate no root but has failed to find the root α as the graph has a turning point at $x = \alpha$.

[3 marks]

Question 4

The diagram below shows the graphs of $y = x$ and $y = \mathrm{g}(x)$.



(a) Show on the diagram, using the value of x_0 indicated, how an iterative process will lead to a sequence of estimates that converge to the x-coordinate of the point P. Mark the estimates x_1 and x_2 on your diagram.

[2 marks]

Question 4

(b) By finding a suitable iterative formula, use $x_0 = 2$ to estimate a root to the equation $x - \sin 0.8x = 2.5$ correct to two significant figures.

[3 marks]

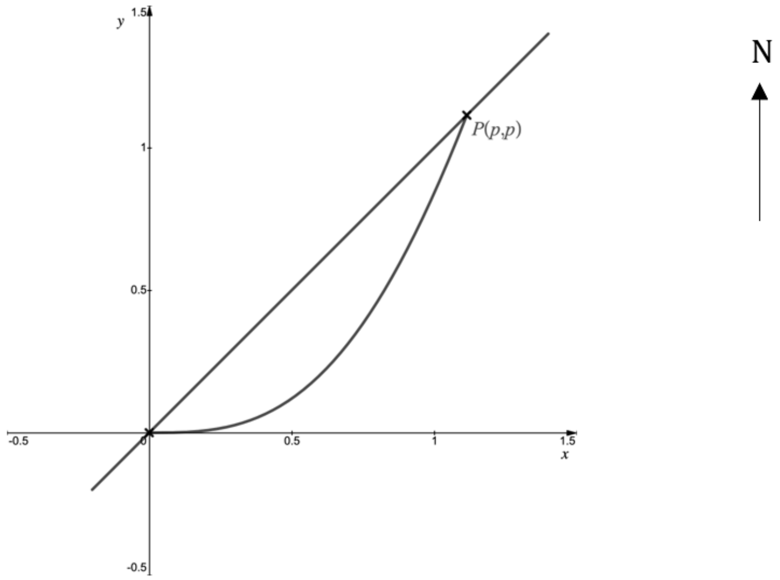
Question 4

(c) Confirm that your answer to part (b) is correct to two significant figures.

[2 marks]

Question 5

The village of Crinkley Bottom lies on a straight road, as modelled by the line $y = x$ on the graph below. Rush hour traffic causes much air pollution in the village so to improve the air quality around Crinkley Bottom a bypass is to be built.
The path of the bypass is modelled by part of the equation $y = x^2 \sin x$.



The bypass is to be built with a roundabout south of the village at the origin and a northern roundabout which re-joins the road through Crinkley Bottom at the point $P(p, p)$.

(a) On the diagram show how using the iterative formula $x_{n+1} = x_n^{\ 2} \sin x$ with $0 < x_0 < p$ will lead to convergence at the southern roundabout

[1 mark]

Question 5

(b) Use the alternative iterative method

$$x_{n+1} = \sqrt{\frac{x}{\sin x}}$$

with $x_0 = 1$, to find the position of the roundabout at P to four significant figures.

[3 marks]

Question 5

(c) Verify that your answer to part (b) is correct to four significant figures.

[2 marks]

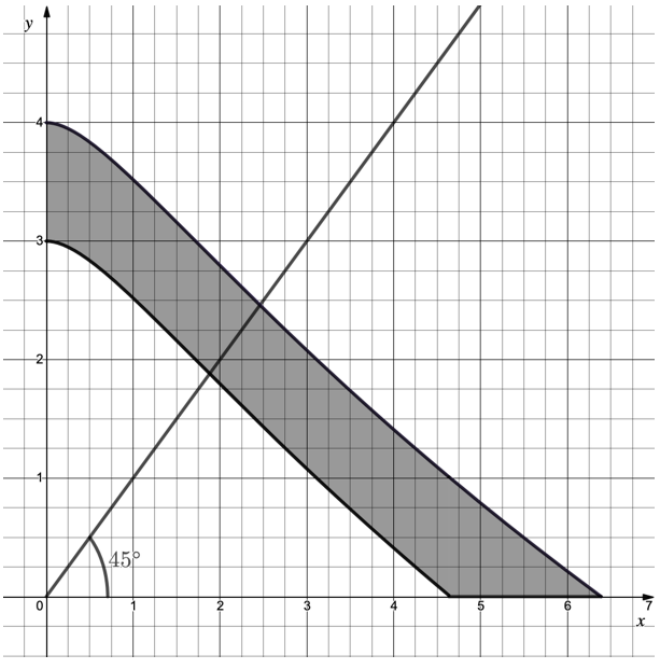
Question 6

The game of Logball is played on a flat table.
A player rolls a ball from a fixed point, at any angle, with the aim of it coming to rest within a winning zone.

A particular player decides to roll the ball at an angle of 45°, as illustrated in the graph below, with the ball being rolled from the origin and the shaded area being the winning zone.

The lower boundary of the winning zone has equation $y = 3 - \ln(x+1)^2 \quad x, y \geq 0$

The upper boundary of the winning zone has equation $y = 4 - \ln(x+1)^2 \quad x, y \geq 0$



(a) Using an appropriate iterative formula with initial value $x_0 = 1.8$, find the minimum distance this player's ball needs to travel to stop within the winning zone. Give your answer to two significant figures.

[3 marks]

Question 6

(b) Using another iterative formula with initial value $x_0 = 2.5$, find the maximum distance this player's ball can travel yet remain within the winning zone.
Give your answer to two significant figures.

[3 marks]

Question 7

According to legend, unicorn tears have magical healing powers.
When a unicorn tear is applied to a bruise of size A mm^2 it will heal according to the model

$$f(t) = Ae^{-0.15t} - 0.2t \qquad\qquad t \geq 0$$

where f is the area of the bruise, in square millimetres, at time t seconds after the unicorn tear is applied.

(a) Show that, for a bruise of initial size 20 mm^2, the equation $f(t) = 0$ can be rearranged into the form

$$t = \frac{20}{3} \ln\left(\frac{100}{t}\right).$$

[3 marks]

Question 7

(b) Using the equation from part (a) as an iterative formula and initial value $t_0 = 12$, find how many seconds it takes a bruise of size 20 mm^2 to heal once a unicorn tear is applied. Give your answer to three significant figures.

[3 marks]

Question 7

(c) It is rumoured that a unicorn tear can heal bruises one-hundred-thousand times faster than they would heal naturally. Approximately how many days would it take a bruise of initial size 20 mm^2 to heal without a unicorn tear?

[2 marks]