6.1 Numerical Methods

Question Paper

Course	Edexcel IAL Maths: Pure 3
Section	6. Numerical Methods
Topic	6.1 Numerical Methods
Difficulty	Easy

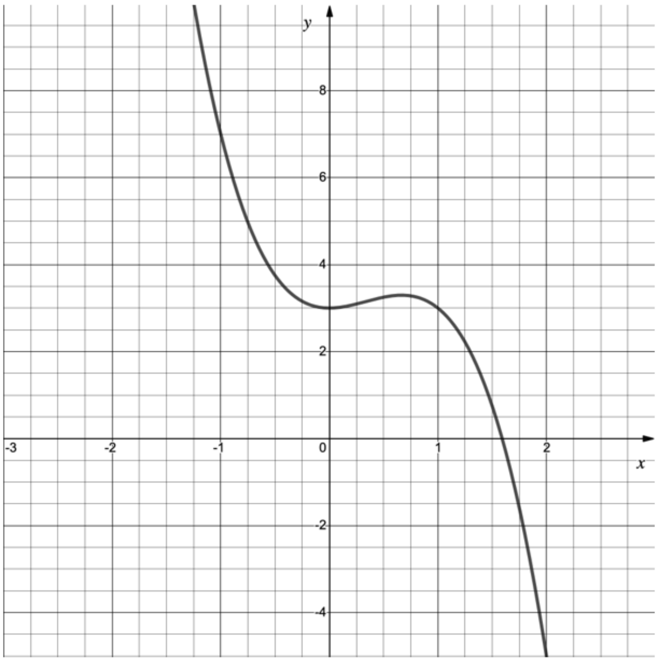
Time allowed: **50**

Score: **/38**

Percentage: **/100**

Question 1

The diagram below shows part of the graph $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = 2x^2 - 2x^3 + 3$.



(a) (i) Find f(1.5)

(ii) Find f(1.6)

[2 marks]

Question 1

(b) Write down an interval, in the form $a < \alpha < b$, such that $\mathrm{f}(\alpha) = 0$, explain clearly your choice of values for a and b.

[3 marks]

Question 2

A solution to the equation $f(x) = 0$ is $x = 3.1$, correct to two significant figures.

(i) Write down the lower bound, l, and the upper bound, u, of 3.1.

(ii) Assuming $f(x)$ is continuous in the interval $l < x < u$, what can you say about the values of $f(u)$ and $f(l)$?

[3 marks]

Question 3

(a) Show that the equation $x^3 - 5x = 2$ can be rewritten as

$$x = \frac{1}{5}(x^3 - 2).$$

[2 marks]

Question 3

(b) Starting with $x_0 = 1$, use the iterative formula

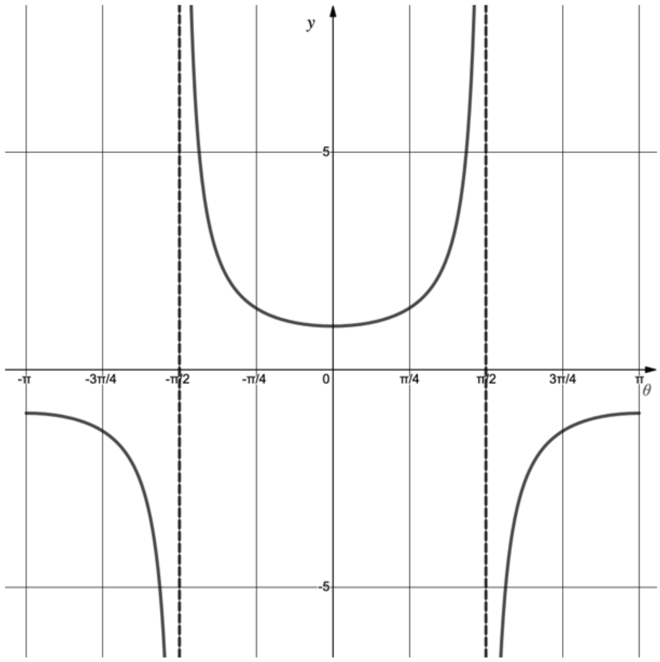
$$x_{n+1} = \frac{1}{5}(x_n^3 - 2)$$

to find values for x_1, x_2 and x_3, giving each to four decimal places where appropriate.

[3 marks]

Question 4

The graph of $y = \mathrm{f}(\theta)$ where $\mathrm{f}(\theta) = \sec\theta$ is shown below. θ is measured in radians and $-\pi \leq \theta \leq \pi$.



Given that $\sec\theta = \frac{1}{\cos\theta}$.

(i) Find $\mathrm{f}(1.5)$ and $\mathrm{f}(1.6)$.

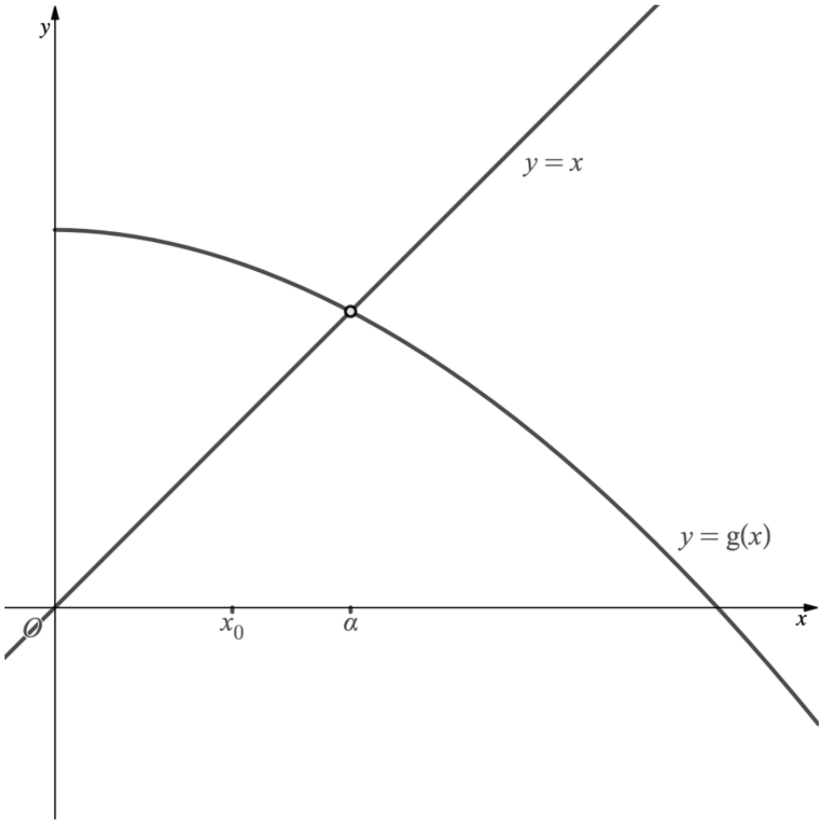
(ii) Explain how, in this case, the change of sign rule fails to locate a root of $\mathrm{f}(\theta)$ in the interval $(1.5\ ,1.6)$.

[3 marks]

Question 5

A student is trying to find a solution to the equation $f(x) = 0$ using an iterative formula.
The student rearranges $f(x) = 0$ into the form $x = g(x)$.

The diagram below shows a sketch of the graphs of $y = g(x)$ and $y = x$.



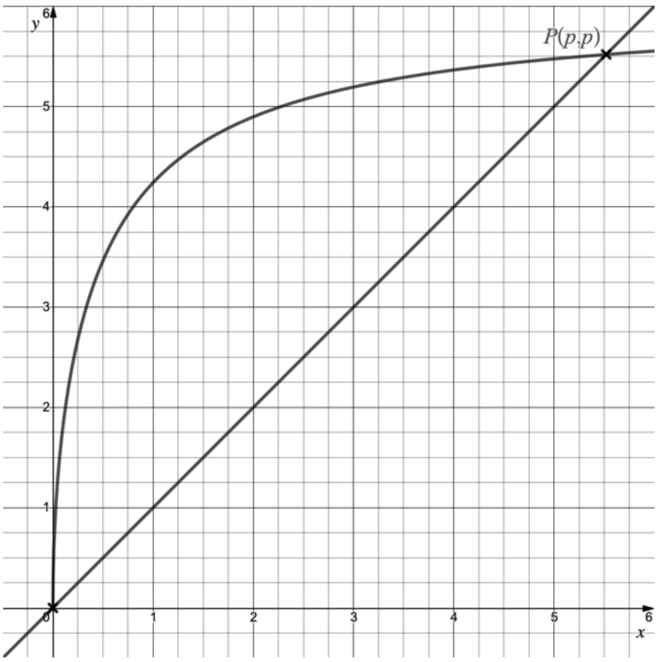
The student is trying to find the root α, starting with an initial estimate x_0.
Show on the diagram, how the iterative formula will converge and find the root α.
Mark the x-axis with the positions of x_1 and x_2.

[3 marks]

Question 6

A bypass is to be built around a village.
On the graph below the road through the village is modelled by the line $y = x$.

The bypass is modelled by the equation $y = \sqrt{\frac{40x}{x+1}}$.



The bypass runs from the origin to the point $P(p, p)$.

(a) Use the iterative formula $x_{n+1} = \sqrt{\frac{40x_n}{x_n+1}}$ with $x_0 = 5$ to find the value of p, correct to three significant figures.

[3 marks]

Question 6

(b) (i) Calculate f(5.835) and f(5.845) where $f(x) = x - \sqrt{\frac{40x}{x+1}}$.

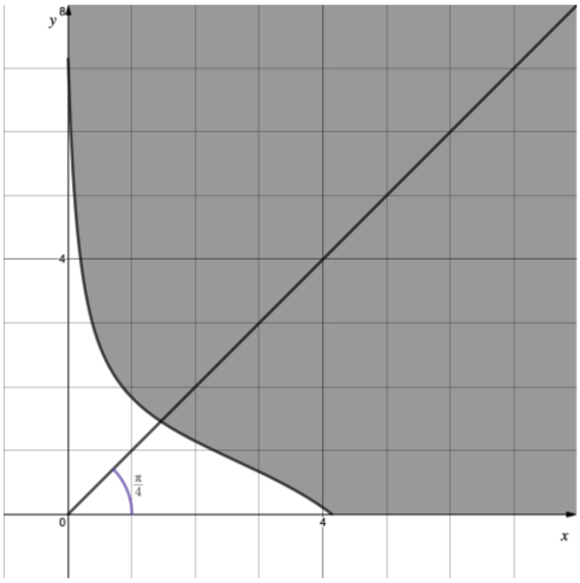
(ii) Hence use the sign change rule to show your answer to part (a) is correct to three significant figures.

[3 marks]

Question 7

The game of Tanball is played on a flat table.
A player rolls a ball from a fixed point, at any angle, with the aim of it coming to rest in the winning zone.

A particular player decides to roll the ball at an angle of $\frac{\pi}{4}$ radians.
This is illustrated by the graph below with the ball being rolled from the origin and the shaded area being the winning zone.



The boundary of the winning zone is given by part of the curve with equation

$$y = 1 - \tan\left(\sqrt{3(x+1)}\right).$$

(i) Using the iterative formula $x_{n+1} = 1 - \tan\left(\sqrt{3(x_n+1)}\right)$, with initial starting value $x_0 = 1.5$, find the estimates x_1, x_2 and x_3, writing each to five decimal places.

(ii) Continue using the iterative feature on your calculator to find the value of x correct to three significant figures.

(iii) Write down the y-coordinate of the point where this player's ball should cross the winning boundary, give your answer to three significant figures.

[4 marks]

Question 8

According to legend, unicorn tears can heal an injury almost instantly.

If a unicorn tear is applied to a burn of initial size B mm^2 on human skin it will heal according to the model

$$\mathrm{b}(t) = B - t^3 + \sqrt{t} \qquad t \geq 0$$

where b is the area of the burn, in square millimetres, at time t seconds after the unicorn tear has been applied.

(a) Show that the equation $\mathrm{b}(t) = 0$ can be written as

$$t = \sqrt[3]{B + \sqrt{t}}.$$

[2 marks]

Question 8

(b) Use the iterative formula $t_{n+1} = \sqrt[3]{40 + \sqrt{t_n}}$, with initial value $t_0 = 3$, to find how many seconds it takes a burn of size 40 mm^2 to heal once a unicorn tear is applied. Give your final answer to three significant figures.

[4 marks]

Question 8

(c) An alternative iterative formula is $t_{n+1} = (t_n^3 - B)^2$.

(i) Using $t_0 = 3$, find t_1, t_2 and t_3, for the same initial burn size as part (b), giving each to three significant figures.

(ii) Explain how you can deduce whether this sequence of estimates is converging or diverging.

[3 marks]